Analysis and Control of Ship Motion in a Random Seaway

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MICHAEL CHASE DAVIS

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MICHAEL CHASE DAVIS

Lieutenant, U. S. Navy

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Signature of Author -
Department of Naval Architecture and Marine Engineering

Certified by -
Thesis Supervisor

Accepted by -
Chairman, Departmental Committee on Graduate Students

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ABSTRACT

This thesis offers a new and fundamental approach to the study of ship motion in a random seaway. The central idea is that the randomness of ship motion is caused by the forces and moments of wave action in the six degrees of freedom of the ship. These six forces constitute a multi-dimensional random process, which can be analyzed with recently developed mathematical theory. The advantages of this approach include: (1) The entire statistics of the sea as they affect ship motion can be determined from measurements made aboard ship, and (2) Theory is available for using the results of the analysis of these forces to develop optimum continuous motion predictions (as would be useful, for example, in an automatic landing system for aircraft carriers) as well as optimum control systems for the simultaneous control of many stabilization power actuators, such as fins or rudders.

In the thesis, mathematical models of the ship are presented in the Laplace transform and in the matrix differential equation forms. The problems associated with correlation function and power spectral density approximations are discussed and contemporary methods analyzed. An original design procedure is developed which will allow simultaneous maximum-effort control of n stabilization devices, minimizing a generalized quadratic error criterion, and which will take account of (1) all statistical interrelations between wave forces, and (2) the inter-coupled nature of ship motion.

Finally, a comprehensive research program is proposed in the areas of ship behavior at sea and ship motion control which emphasizes the use of multi-dimensional random process theory as a basic tool.

Thesis Supervisor: J. R. Baylis

Title: Associate Professor of Naval Engineering

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CHAPTER I.

A NEW APPROACH TO SHIP MOTION

1.1 Introduction

This thesis is concerned with a new approach to the understanding of the motions of ships in a seaway. A unified theory will be established which is heavily dependent on the methods used by the control systems engineer, who has spearheaded much of the contemporary advances in the study of complex dynamical systems.

Complexity is the necessary penalty for an adequate analysis of the interdependent motions of a ship in six degrees of freedom. With the advent of high-speed, large capacity digital computing equipment, complexity and dimensionality alone are not formidable deterrents to understanding, providing that the analyst has convenient methods for expressing the relations involved and has a well-defined objective for analysis.

This work is to be considered as a practical application of general theory developed in a companion volume[1], the latter being motivated by the dearth of theoretical aids to understanding of the random nature of total ship motion. Thus a major goal of this thesis is to interpret and to illustrate the practical application of the mathematical concepts presented in Ref. 1.

The hoped-for result of this work is to outline a new approach to ship motion which will yield dividends in more complete knowledge of the properties of the ship and of the sea, and what is perhaps just as important, will provide rational design procedures for control systems which tend to constrain or control ship motion.

1.2 Current state-of-the-art

A very useful summary of contemporary knowledge on the subject

of ship motion is found in the monumental work of Korvin-Kroukovsky[2], who has made a critical evaluation of most of the facets of past research on the sea and its effect on ships. It would be pointless to attempt to parrot this work to any extent, but there are many results of recent research which tend to support the approach to be presented in this volume.

(a) The hypothesis of linearity

First of all, consider the ship at sea, subject to hydrostatic and hydrodynamic forces which cause complex motions of the rigid structure. Conventionally, this motion is resolved into translations along three normal axes of a three-coordinate system and rotations about these axes. As will be illustrated in Section 2.2, these motions can be considered as the solution of a six-fold set of intercoupled second-order differential equations, with the sea providing the exciting forces. It is well-known that most systems which occur in nature can be approximated in a sufficiently small region of validity by linear differential equations, regardless of strong non-linearities in other regions. The question which naturally arises is, can the ship be adequately described by linear differential equations in its normal operating range of motion at sea?

Despite the prolific investigations of various non-linearities of ship motion which have appeared in the literature, it appears that the linear approximation is valid in many cases for the actual ship at sea. To support this hypothesis, model investigations made by Korvin-Kroukovsky and Jacobs[3], and later by Gerritsma[4] in regular waves showed good agreement between experiment and linear theory. Lewis[5], Lewis and Numata[6], and Lewis and Dalzell[7] conducted towing tank tests in irregular seas which verified the principle of superposition, the consequence of linearity, for the motion of the model with respect to wave components at various frequencies. Also, in Section 3.2 of this work, statistical properties of motion measured aboard the USS GYATT (DDG 1) in February 1959 by Marks and Durkovic[8] will be shown to agree nicely with a linear hypothesis.

As an aside, it is indeed fortunate that the blessings of linearity can be attributed to the ship at sea, for a rigorous theory for non-linear analysis of complex dynamic devices is in its infancy, and when stochastic processes are allowed as an excitation force, the understanding of such a problem is many years away.

(b) Description of the sea and ship motion as a random process

The sea defies a determinate description. Its ever-changing surface with waves rushing here and there, combining, disappearing -- this would appear to be a phenomenon which is absolutely indescribable in the large. However, some progress has been made in delineating the nature of the sea with statistical methods.

The fundamental building block for contemporary statistical study of the sea is the unidirectional regular wave. If superposition is assumed to apply, which regards the air-water interface as a linear transmission medium, the sea can be considered to be constructed of a distribution or spectrum of waves of various amplitudes, wavelengths, phases, and directions. Since the sine wave has the virtue of orthogonality, has been conventionally used in the description of random processes in other fields, and is an approximation to the waves which propagate naturally at sea, it naturally is used as the fundamental wave in the superposition procedure.

Some of the major contributors to this ever-expanding body of knowledge have been St. Denis and Pierson[9], Neumann[10], Darbyshire[11], Voznessensky and Firsoff[12], Barber[13], Longuet-Higgins[14], and Marks[15].

Longuet-Higgins[14] has made what is perhaps the most appealing derivation of the description of the sea. He shows that a generalized autocorrelation function of the sea can be expressed by

$$\varphi(x, y, t) = \lim_{X, Y, T \to \infty} \frac{1}{XYT} \int_{-X}^{X}\int_{-Y}^{Y}\int_{-T}^{T} N(x', y', t')\, N(x'+x, y'+y, t'+t)\, dx'\, dy'\, dt' \quad (1.1)$$

Here, N is the wave height at a point, x and y are normal position coordinates, and t is a time variable.

Unfortunately, there is no known theory which would permit manipulation of the above expression or its possible transform mates (spectra) -- provided that they could even be measured -- in order to reproduce useful results in the study and control of ship motion.

Neumann[10] has had a considerable influence in the theoretical and practical development of sea spectral theory. By combining a shrewd analysis of early experimental data with the dynamics of waves at sea, he developed an expression for the spectrum (or frequency content) of the wave-height at a point

$$\Phi(\omega) = \frac{K_1}{-\omega^6} \, e^{-\frac{K_2}{\omega^2}}$$

where the K_i terms indicate constants.

St. Denis and Pierson[9] hypothesized a directional sea spectra of

$$N(\omega, \theta) = \frac{K_3}{-\omega^6} \, e^{-\frac{K_2}{\omega^2}} \cos^2\theta$$

where θ represents the direction of wave travel as measured from a reference direction of maximum wave travel.

They proposed that if measurements were made on a ship model to determine the response of the ship, both amplitude and phase, to a sinusoidal wave excitation which varied over a range of frequencies and a range of oblique directions to the direction of motion, the response of the ship could be calculated by weighting each response function by the amount of excitation at that frequency and from that direction, and summing the total. This idea has had a mjaor effect in shaping current ideas about ship response in irregular waves.

An alternate approach to the problem of defining wave spectra at a point was presented by Voznessensky and Firsoff[12]. They made experimental measurements of wave auto-correlation functions, which are

the inverse Laplace transforms of wave spectra, and found that they could all be approximated by an exponentially-damped cosine wave, which will be later shown to agree nicely with a linear hypothesis.

Finally, it is necessary to consider how the random process of wave motion arose in the first place. A number of investigations have been made into the mechanism of energy transfer from wind gusts to wave motion. According to Korvin-Kroukovsky[2], this problem is essentially unsolved. From basic considerations, however, the wave action that a ship sees is generally a result of random wind action over great areas of the earth's surface and over long periods of time.

(c) Summary

It is useful to summarize at this point the major facts from contemporary theory which will be of use in the presentation to follow.

(1) The ship is in general suitably described by the assumption of linearity.

(2) The equations most used for "theoretical" wave spectra are actually based on an analysis of early experimental data.

(3) The usual approach in expressing motion of a ship in a random seaway is to consider it as the effect of a wave spectra exciting a linear system whose continuum of transfer functions can be determined by experiment from ship model response to oblique waves.

(4) Correlation functions of wave records show a tendency to approximate an exponentially decaying cosine wave.

(5) The actual cause of the waves observed by a ship are a very large number of essentially independent wind gusts, the effect of each making a very small contribution to the wave height observed at a single point.

1.3 Outline of a new approach to the ship motion problem

The heart of the contemporary approach to ship motion in irregular seas is presented in Figure 1.1. Here the wind is shown as a disturbing variable acting on the sea to create a wave spectrum. The wave spectrum, in turn, acts on a ship to produce motion. The original cause, the wind, is such a random phenomenon and is distributed over such a large area

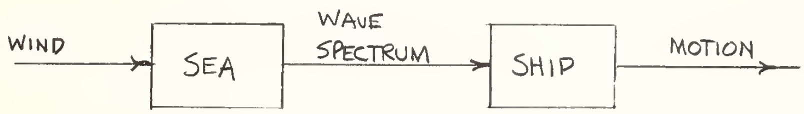


Fig. 1.1 Usual approach to ship motion analysis

that any attempt to measure its effect would be ludicrous. Therefore, in the large, the system or mechanism which converts the total wind disturbance to a sea spectrum is forever unknowable. The spectrum itself is capable of measurement, but the only full scale attempt to measure it at sea, Project SWOP[16], was a very drawn out, complicated, and expensive operation to obtain the statistical properties of a small patch of ocean at a certain instant of time. The continuum of transfer functions which relate the sinusoidal response of the ship to an oblique wave can be measured in the towing tank, but this would involve a very elaborate testing program because of two variables involved, direction and frequency. It appears that the study of ship motion, by focussing attention on the causal action of a sea spectrum which is in practice unmeasurable, has painted itself into a corner. This corner is the stagnation which must necessarily result in a theory which depends on parameters which are so difficult to establish.

Suppose that a return to the most fundamental principals is made. The motion of a ship at sea is mathematically approximated by a system of constant-coefficient differential equations. In these equations appear a forcing function of six dimensions -- the forces along and moments about the three coordinate axes. These forces represent the effects of the random seaway to the ship. Fig. 1.2 shows the sequence of actions considered.

Fig. 1.2 A new approach to ship motion analysis

Let us assume, for the moment, that the equation coefficients of a particular ship are known. This had been the goal of a great deal of theoretical and experimental work in the past decade, and at this writing, it appears that a reasonable approximation could be made on the basis of theoretical analysis and model experiments for a given ship. Knowing these coefficients, it would be entirely within the scope of current technology to measure the displacement, velocity, and acceleration variables of the various degrees of freedom of the ship, weigh these numerically by their respective coefficients in the differential equations, and compute the effective forces on the ship. That is, we would continuously produce on a ship at sea a record of the approximate six force variables which would be composed of suitable weighted values of measured ship motion. This six-fold record would be a random process, a so-called multi-dimensional random process. Providing that our approximations to the equations of the ship are valid, we would have a record of the total random variation of the sea as it has affected the ship over the interval of time considered. It is important to note that the ship "system" considered here includes the hydrostatic and hydrodynamic forces which would act on the ship if it were suddenly released from an out-of-equilibrium condition in an otherwise smooth sea. The random force inputs, then, are the integrated superimposed effects of the wave system acting on the ship.

It is contended therefore, and this is the most important statement in this work, that the fundamental goal of the analysis of ship motion at sea should be to investigate the statistical nature of these wave forces on the ship.

The logical question for the reader to ask at this point is -- How does one analyze a record of six simultaneous inter-related random processes

to extract useful information from it? A theoretical answer to this question was presented by the author in a companion document[1] which was essentially motivated by the particular problem of analyzing ship motion at sea.

Some of the advantages to be expected from pursuing this line of investigation are:

(1) A catalogue of sea descriptions can be built up for a ship class.

(2) All statistical data used can be collected aboard ship.

(3) A quantitative means for determining the effect of hull coefficient changes on ship motion in typical seas will result.

(4) A means of continuously making an optimum short-time prediction of any or all components of future motion is provided.

(5) Based on this approach, a design procedure is possible which yields near-optimum control of steering and stabilization devices, allowing (1) for saturation of the force transducers, (2) for the intercoupling of motions in the ship, (3) for the statistical relations among the various wave-produced forces and moments, and (4) for the possibility of noise-corrupted information from the motion sensors.

The following chapters will develop and elaborate on the theory and practical use of this new approach to the understanding and control of shipboard motion.

CHAPTER II.

MATHEMATICAL MODELS FOR THE SEA AND SHIP

2.1 Mathematical tools required

The study of such a complex phenomenon as ship motion at sea is not suitably undertaken with horse-and-buggy mathematical methods. In fact, the success of such an analysis will be highly dependent on methods which can convey large ideas with a minimum of notational difficulty.

The first major problem which must be resolved is the most efficient handling of problems involving the dynamic behavior of linear systems. Experience in the fields of control and communications engineering has shown that the most useful expression for linear systems analysis is the transfer function in terms of the Laplace transform variable, s. Its use avoids the unpleasant task of convolution when the response of a system is to be determined from knowledge of its excitation. Suppose a transfer function G(s) of a system is given by

$$G(s) = \frac{Z(s)}{P(s)}$$

where Z(s) and P(s) are polynomials in s. G(s) is the ratio of the Laplace transform of the output to that of the input. The easiest way of visualizing the meaning of this expression is to consider the differential equation which effectively relates the input x and output y of the system. If $\frac{d^n}{dt^n}$ is replaced by the operator s^n, then this differential equation will have the form

$$P(s)\ y = Z(s)\ x$$

Formally, the definition of the Laplace transform G(s) of a function of time g(t) is given by

$$G(s) = \int_0^{\infty} g(t)\ e^{-st}\ dt \tag{2.1}$$

Throughout the literature of naval architecture and, in particular,

ship motion, the "jω" notation is used in place of the fundamental transform variable, s, to designate the transfer function of a physical system. Superficially, this would seem to be a difference of small importance, but in practice it has demonstrated the extreme preoccupation of contemporary theorists with the ideas of simusoidal response of systems, a point of view which it is hoped will suffer in the treatment to follow.

The second major mathematical tool which must be brought into play is matrix theory, often called the algebra of higher mathematics. The use of matrices permits handling, at least conceptually, dynamic linear problems of arbitrary complexity. With six degrees of freedom of the ship, matrix methods are a necessity.

It is assumed in the following that the reader is familiar with the basic elements of Laplace transform and matrix theory.

2.2 Equations of motion of the ship

In this section the equations which govern the rigid-body motion of a ship in its six degrees of freedom will be reviewed, and manipulated into a form most suitable for analysis. The conventional derivations of the equations of motion will not be presented here, as they are available elsewhere[17], but some motivation for their appearance will be given.

By equating the vector force $\vec{F}$ on a ship to the time rate of change of momentum,

$$\vec{F} = \frac{d}{dt} (m \vec{V}) = m\left\{\frac{d}{dt}|\vec{V}|\right\}\frac{\vec{V}}{|V|} + \vec{\Omega} \times m\vec{V}$$

and the vector moment $\vec{Q}$ to the time rate of change of angular momentum

$$\vec{Q} = \frac{d}{dt} (\overrightarrow{I\Omega}) = \left\{\frac{d}{dt}|\overrightarrow{I\Omega}|\right\}\frac{\overrightarrow{I\Omega}}{|I\Omega|} + \vec{\Omega} \times \overrightarrow{I\Omega}$$

a general set of equations results which gives the three-dimensional motion of a free body under the action of external forces. Here, $\vec{V}$ is the instantaneous velocity of the ship, $\vec{\Omega}$ is the instantaneous angular velocity, and $\overrightarrow{I\Omega}$ is the

angular momentum vector.

A suitable right-handed orthogonal coordinate system is selected:

(1) Under equilibrium conditions at constant speed U_o, x is fixed in the ship from the center of gravity in the direction of forward motion, y is to starboard, and z is down.

(2) The angular displacements -- ϕ (roll), θ (pitch), and ψ (yaw) -- are measured in a clockwise sense from the equilibrium positions about the x, y, and z axes, respectively.

(3) Linear velocities along and angular velocities about the x, y, and z axes are denoted by u, v, w and p, q, r, respectively.

With this choice of coordinates, six equations result which relate the force or moment in a given direction to the resulting motion:

$$
\begin{aligned}
F_x &= m(\ddot{x} + qw - vr) \\
F_y &= m(\ddot{y} + ru - pw) \\
F_z &= m(\ddot{z} + pv - qu) \\
M_x &= I_x \dot{p} + qr(I_z - I_y) \\
M_y &= I_y \dot{q} + rp(I_x - I_z) \\
M_z &= I_z \dot{r} + pq(I_y - I_x)
\end{aligned}
$$

These equations are linearized about the equilibrium point, yielding

$$
\begin{aligned}
F_x &= m\ddot{x} \\
F_y &= m\ddot{y} + mU_o r \\
F_z &= m\ddot{z} - mU_o q \\
M_x &= I_x \dot{p} \\
M_y &= I_y \dot{q} \\
M_z &= I_z \dot{r}
\end{aligned}
$$

where U_o is the mean forward speed of the ship. The other terms involving the product of two velocities disappear because of their zero equilibrium values.

Next, the hydrodynamic and hydrostatic forces exerted by the mo-

tion of the ship are examined. Initially, on a body of arbitrary shape it can be assumed that accelerations, velocities, and displacements in or about any axis cause forces in or about any other axes. This would indicate that 108 coefficients in general would be needed to specify the system, a staggering thought. A considerable simplification results in considering the real ship:

First of all, linear displacements in the x or y directions could not be expected to result in a force on the ship. Next, the symmetry of the usual ship form about its centerline plane prevents small motion in the vertical plane (heave, pitch, and surge) from inducing forces in other degrees of freedom. Also, the surge or motion in the direction of mean forward velocity can sensibly be regarded as being decoupled from other motions.

The linear differential equations which result from these considerations are certainly approximations to the actual case -- the hydrostatic and hydrodynamic forces are in general non-linear, the coordinate system used is valid only in a certain range near equilibrium, and the assumption that the wave excitation forces occur independently of the motion of the ship is certainly debatable. With all these misgivings, we still plunge ahead and trust that the resulting theory will provide (1) useful results for a ship at sea and (2) a foundation for a more complete theory as inadequacies are revealed by future experimental results.

The equations of motion are conveniently expressed in Laplace transform notation, using matrices,

$$\underline{H(s)} \quad v(s)\Big] = F_c(s)\Big] + F_\omega(s)\Big] \tag{2.2}$$

Here $\underline{H(s)}$ is a 6x6 matrix which expresses the dynamic forces which act on the ship as a result of its motion. $V(s)\Big]$ is a six-dimensional vector of the displacements of the ship from equilibrium. $F_c(s)\Big]$ and $F_\omega(s)\Big]$ indicate the forces of controlled elements and waves, respectively, in each of the degrees of freedom.

To structure these equations, the following components make up the displacement vector v. They are grouped in the order given in order to emphasize the interacting motions.

v_1 --Roll, ϕ ; v_2 --Yaw, ψ ; v_3 --Sway, y; v_4 -- Heave, z; v_5 -- Pitch, θ ; and v_6 -- Surge, x.

H(s) is constructed from three sets of intercoupling motions -- $H_1(s)$, roll-yaw-sway; $H_2(s)$, heave-pitch; and $H_3(s)$, surge.

$$H(s) = \left[\begin{array}{c|c|c} H_1(s) & 0 & 0 \\ \hline 0 & H_2(s) & 0 \\ \hline 0 & 0 & H_3(s) \end{array}\right] \qquad (2.3)$$

$$H_1(s) = \begin{bmatrix} s^2(K_{\dot{p}} + I_x) + sK_p + K_\phi & s^2K_{\dot{r}} + sK_r & s^2K_{\dot{v}} + sK_v \\ s^2N_{\dot{p}} + sN_p & s^2(N_{\dot{r}} + I_z) + sN_r + N_\psi & s^2N_{\dot{v}} + sN_v \\ s^2Y_{\dot{p}} + s(Y_p + mU_0) & s^2Y_{\dot{r}} + sY_r + Y_\psi & s^2(Y_{\dot{v}} + m) + sY_v \end{bmatrix} \qquad (2.4)$$

$$H_2(s) = \begin{bmatrix} s^2(Z_{\dot{w}} + m) + sZ_w + Z_z & s^2Z_{\dot{q}} + s(Z_q - mU_0) \\ s^2M_{\dot{w}} + sM_w + M_z & s^2(M_{\dot{q}} + I_y) + sM_q + M_\theta \end{bmatrix} \qquad (2.5)$$

$$H_3(s) = \left[s^2(m + X_{\dot{u}}) + sX_u \right] \qquad (2.6)$$

The components of forces or moments in F_c and F_ω follow the order of V above.

This set of equations is believed to be one possible approximate representation of the linear model for the ship, but additions or simplifications will not affect the general validity of the theory to follow in future sections. The coefficients in the elements are expressed in standard notation[17], with the important exception of a reversal in sign. It is strongly felt that general coefficients of motion used in differential equations should be positive, reserving the negative sign as an indicator of such particular phenomenon as negative damping or a negative spring constant, which are usual sign-posts of instability.

Solving Eq. 2.2 for the motion of the ship,

$$\left.V(s)\right] = \underline{H^{-1}(s)} \left\{ \left. F_c(s) \right] + \left. F_\omega(s) \right] \right\} \tag{2.7}$$

where

$$\underline{H^{-1}(s)} = \left[\begin{array}{c|c|c} H_1^{-1}(s) & 0 & 0 \\ \hline 0 & H_2^{-1}(s) & 0 \\ \hline 0 & 0 & H_3^{-1}(s) \end{array} \right] \tag{2.8}$$

Figure 2.1 shows the ship as a "system", being excited by wave and control forces.

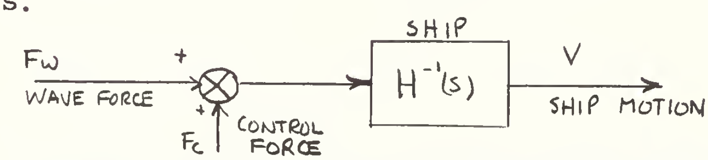


Fig. 2.2 General model for multi-dimensional ship motion

Besides the transform approach, there is an alternate way of characterizing linear systems which is appealing in its generality and compactness of notation. This is by means of the so-called matrix differential equation, as found in, for example, Bellman[18]. In this method, the primary quantity of interest is not the input or output, but rather the state variables of the system. These are a set of quantities which, if specified as initial conditions, would be sufficient to determine tne entire free response of the system as it settled to equilibrium.

To motivate this approach, consider the simple first order differential equation

$$\frac{d}{dt} x(t) = a\, x(t) + d\; c(t)$$

where a and d are constants, and c(t) is an independent variable. If c(t) is zero, and x(0) is specified, the solution of the equation is

$$x(t) = x(0)\, e^{a\,t} \qquad (t \geq o)$$

Now, suppose we write the same equation in matrix notation.

$$\frac{d}{dt} x(t) = A x(t) + D c(t) \tag{2.9}$$

where A and D are matrices with numerical elements, and x(t) and c(t) are state and control vectors, respectively. If c(t) is zero, and x(0) is known, the solution of this equation is

$$x(t) = e^{At} x(o) \qquad (t \geq o) \tag{2.10}$$

from the theory of matrix differential equations. e^{At} is an nxn matrix, known as the matrix exponential, which is defined by

$$e^{At} = \sum_{i=0}^{\infty} A^{i} \frac{t^{i}}{i!}$$

The ability to express the initial condition response in such a simple form for linear systems of arbitrary complexity is a considerable advantage to the analyst.

The ship equations of motion will now be cast into the matrix differential form, and the numerical elements of the matrices will be identified as functions of the ship coefficients. Also, the state variables of the ship will be defined. In the following sections the concept of state and initial condition decay will play a prominent part.

From Eq. 2.2,

$$\underline{H(s)} \; V(s)\rfloor = F_c(s)\rfloor + F_{\omega}(s)\rfloor \tag{2.2}$$

H(s) can be expressed as a sum of three matrices

$$\underline{H(s)} = s^2 \underline{M} + s \underline{B} + \underline{R} \tag{2.11}$$

where

$$\underline{M} = \begin{bmatrix} K_{\dot{p}} + I_x & K_{\dot{r}} & K_{\dot{v}} & 0 & 0 & 0 \\ N_{\dot{p}} & N_{\dot{r}} + I_z & N_{\dot{v}} & 0 & 0 & 0 \\ Y_{\dot{p}} & Y_{\dot{r}} & Y_{\dot{v}} + m & 0 & 0 & 0 \\ 0 & 0 & 0 & Z_{\dot{\omega}} + m & Z_{\dot{q}} & 0 \\ 0 & 0 & 0 & M_{\dot{\omega}} & M_{\dot{q}} + I_y & 0 \\ 0 & 0 & 0 & 0 & 0 & X_{\dot{u}} + m \end{bmatrix} \tag{2.12}$$

$$\underline{B} = \begin{bmatrix} K_p & K_r & K_v & 0 & 0 & 0 \\ N_p & N_r & N_v & 0 & 0 & 0 \\ Y_p + m\,U_o & Y_r & Y_v & 0 & 0 & 0 \\ 0 & 0 & 0 & Z_w & Z_q - m\,U_o & 0 \\ 0 & 0 & 0 & M_w & M_q & 0 \\ 0 & 0 & 0 & 0 & 0 & X_u \end{bmatrix} \tag{2.13}$$

$$\underline{R} = \begin{bmatrix} K_\phi & 0 & 0 & 0 & 0 & 0 \\ 0 & N_\psi & 0 & 0 & 0 & 0 \\ 0 & Y_\psi & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Z_z & Z_\theta & 0 \\ 0 & 0 & 0 & M_z & M_\theta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{2.14}$$

Rearranging Eq. 2.11 and substituting the expression for $\underline{H(s)}$ into Eq. 2.2, $s^2 \underline{M}\, V(s)\rfloor = -s\,\underline{B}\, V(s)\rfloor - \underline{R}\, V(s)\rfloor + F_c(s)\rfloor + F_\omega(s)\rfloor$

Multiplying each side by $\underline{M}^{-1}$

$$s^2 V(s)\rfloor = -s\,\underline{M}^{-1}\underline{B}\, V(s)\rfloor - \underline{M}^{-1}\underline{R}\, V(s)\rfloor + \underline{M}^{-1} F_c(s)\rfloor + \underline{M}^{-1} F_\omega(s)\rfloor \tag{2.15}$$

The following state variables are defined:

x_1 = Roll velocity, p
x_2 = Yaw velocity, r
x_3 = Sway velocity, v
x_4 = Heave velocity, w
x_5 = Pitch velocity, q
x_6 = Surge velocity, u
x_7 = Roll displacement, ϕ
x_8 = Yaw displacement, ψ
x_9 = Heave displacement, z
x_{10} = Pitch displacement, θ

The following identifications can be made immediately.

$s^2\, V(s)\rfloor = s\,X_i\rfloor \qquad i = 1, 2, \ldots 6$

$s\, V(s)\rfloor = X_i\rfloor \qquad i = 1, 2, \ldots 6$

Considering the product $\underline{R}V(s)\rfloor$ it is seen that this is equivalent to

$$\underline{R}\, V(s)\Big] = \begin{bmatrix} K_\phi & 0 & 0 & 0 \\ 0 & N_\psi & 0 & 0 \\ 0 & Y_\psi & 0 & 0 \\ 0 & 0 & Z_z & Z_\theta \\ 0 & 0 & M_z & M_\theta \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_4 \\ V_5 \end{bmatrix} \triangleq \underline{K} \quad X_j\Big]$$

$$j = 7, 8, 9, 10$$

Also, by definition,

$$s \begin{bmatrix} X_7 \\ X_8 \\ X_9 \\ X_{10} \end{bmatrix} = \begin{bmatrix} X_1 \\ X_2 \\ X_4 \\ X_5 \end{bmatrix}$$

Assembling these equations into a single matrix equation, with $\underline{0}$ indicating a matrix of all zero elements,

$$\frac{d}{dt}\, x\Big] = \left[\begin{array}{c|c} -M^{-1}B & -M^{-1}K \\ \hline \begin{matrix} 1&0&0&0&0&0 \\ 0&1&0&0&0&0 \\ 0&0&0&1&0&0 \\ 0&0&0&0&1&0 \end{matrix} & 0 \end{array}\right] x + \left[\begin{array}{c} M^{-1} \\ \hline 0 \end{array}\right] F_c + \left[\begin{array}{c} M^{-1} \\ \hline 0 \end{array}\right] F_W \qquad (2.16)$$

or, in general form,

$$\frac{d}{dt}\, x = A\, x + D_c\, F_c + D_W\, F_W \qquad (2.17)$$

with the numerical matrices A, D_c, and D_W being defined above as a function of the coefficients of the equations of motion.

To summarize, the ship has been described in terms of a Laplace transform matrix, H(s), and a numerical matrix A-- the two basic descriptions of linear systems --in terms of the physical parameters and

hydrodynamic coefficients of ship motion. This will permit the efficient discussion of ship motion in the following sections.

2.3 Important relations in scalar random process theory

The purpose of this section will be to present and analyze some pertinent relations in the study of random processes and their effects on linear systems. The emphasis here will be on viewpoint, and the reader is referred to standard texts[19] or Ref. 1 for proofs and extensions.

Statistical analyses of random processes are made in order to detect relations which exist between signals at separated instants of time. The class of random processes considered here are stationary -- the statistics do not vary with time -- and satisfy the ergodic hypothesis, which essentially means that the desired statistics can be adequately approximated through analysis of signals over a long interval of time. This theory becomes more applicable as the process considered approaches a Gaussian random process -- which latter case is approximately true in the ship motion problem.

The fundamental measure of random processes is the correlation function, $\varphi(\tau)$, defined by

$$\varphi_{xy}(\tau) \;=\; E\left\{x(t)\;.\;y(t+\tau)\right\} \qquad (2.18)$$

where x(t) and y(t) are two not necessarily distinct random processes, and $E\{.\}$ indicates the expected or mean value. When x and y are different, $\varphi_{xy}(\tau)$ is known as a cross-correlation function, and when y is identical to x, $\varphi_{xx}(\tau)$ is an auto-correlation function. Fig. 2.2 shows typical correlation functions.

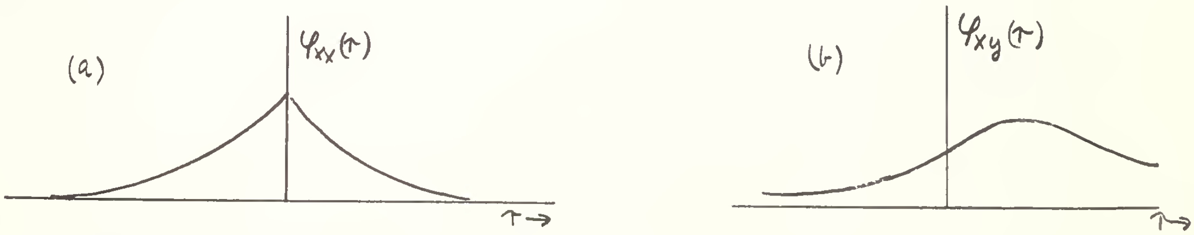


Fig. 2.2 (a) Typical auto-correlation function. (b) Typical cross-correlation function.

Since correlation functions (for $\tau > 0$) have the appearance of transient signals in a linear system -- that is, decaying to zero as $\tau \rightarrow \infty$ -- it is natural to use their Laplace transform as an alternate measure of correlation. These transforms are known as auto or cross power density spectra. Since correlation functions are defined for positive and negative time, the bilateral or two-sided Laplace transform is conventionally selected[20].

If $j\omega$ is substituted for the Laplace transform variable s, the auto power density spectrum can have the connotation of an amplitude-squared distribution of sinusoidal frequency content in a signal except for a constant. For a cross power density spectrum in $j\omega$, the information is conventionally represented in naval architecture literature by co and quadrature spectra, which are its real and imaginary parts, respectively.

It is the contention of the present author that preoccupation with the frequency or ω representation of random processes tends to stifle progress in the understanding of random processes, especially when cross power density spectra are considered. For example, it is impossible to acquire much feel for what is actually happening between a pair of random processes from consideration of their co and quadrature spectra, yet these observations have appeared quite frequently in recent literature.

As a more basic statement of the worth of power density spectra in the analysis of random processes, this thesis will demonstrate that their main use should be as a stepping-stone in determining the transfer functions of linear systems.

The general formula which governs the change in statistical properties of signals after passage through a linear system is

$$\Phi_{xy}(s) = \sum_{i=1}^{m} \sum_{j=1}^{m} G_i(-s)\, H_j(s)\, \Phi_{x_i y_j}(s) \qquad (2.19)$$

Here, $\Phi_{xy}(s)$ is a cross power density spectrum between signals x and y (they may be identical) or the Laplace transform of $\varphi_{xy}(\tau)$. x is made up of n signals, x_i, each passing through a linear system with trans-

fer function $G_i(s)$, and y is similarly composed of m y_j signals, each of which have been operated on by $H_j(s)$. The set of input statistics

$$\Phi_{x_i y_j}(s) \quad (i = 1, 2 \ldots n; \quad j = 1, 2 \ldots m)$$

is assumed known in order to completely specify x and y.

As a trivial example of the application of this formula, consider Fig. 2.3, where the input spectra of x, $\Phi_{xx}(s)$, is specified, and the output spectra $\Phi_{yy}(s)$ and the cross spectra between x and y, $\Phi_{xy}(s)$, are to be found.

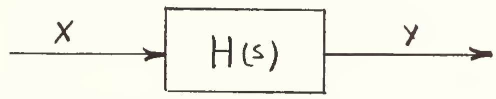


Fig. 2.3 A simple linear system

Since $x_1 = x$, $G_1 = 1$; $y_1 = x$ and $H_1(s) = H(s)$,

$$\Phi_{yy}(s) = H(-s)\ H(s)\ \Phi_{xx}(s)$$

and

$$\Phi_{xy}(s) = H(s)\ \Phi_{xx}(s)$$

To make an application of this result which will be of continuing significance in future sections, suppose that the input signal x has an auto-correlation function which is a unit impulse at the origin. Since $\varphi_{xx}(0) = E\left\{x^2(t)\right\}$, this impulse would mean an infinite mean-square value which is, of course, not possible. However for many practical problems the assumption of an impulse is valid, just as in mechanical problems a force impulse is used to approximate a delivery of momentum over a small finite interval. This signal is known as white noise, and in the transfer domain, has a constant value at all values of ω . It is observed, for example, as "grass" on a radar scope. The important thing about white noise is that is the ultimate in randomness and is totally uncorrelated with time-shifted versions of itself. Also, it can be approximated very nicely for use in analog simulation.

Thus if $\Phi_{xx}(s) = 1$, then

$$\Phi_{yy}(s) = H(-s)\,H(s)$$

It is known that power density spectra of single random processes expressed as rational functions of s can always be factored into a form

$$\Phi(s) = H(-s)\,H(s)$$

Thus, a random process can always be viewed as the output of a linear system, denoted by H(s), which is excited by white noise. By postulating the totally random excitation, one can focus attention on the effective linear system rather than on the statistics of the random processes. This viewpoint is not just an abstraction, for an hypothesis has been offered in the literature[21] that most random phenomena observed can be traced back to an original cause which has the properties of almost-white noise.

An appreciation for the strength of this approach can be obtained through considering two fundamental problems of random process theory --the optimum filter and the optimum predictor. These new results were first presented in Ref. 1, and illuminate the basic mathematical solutions given in the classical work of Wiener[22].

a. The optimum filter problem

Figure 2.4 shows a linear system W(s) which must operate on a random process v. The signal v is assumed to consist of signal s and noise n, with the criterion for optimum performance being the minimiza-

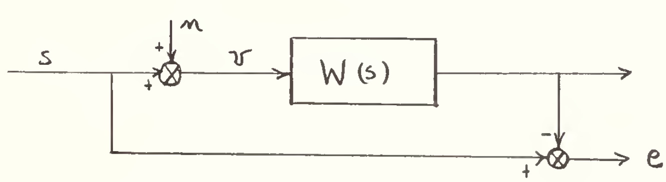


Fig. 2.4 The basic filtering probl em

tion of the mean square error between output and signal, $\overline{e^2}$.

If noise and signal are uncorrelated,

$$\Phi_{vv}(s) = \Phi_{ss}(s) + \Phi_{nn}(s)$$

from Eq. 2.19. Let us assume that the factors of the denominators of each term, the poles, are distinct. Suppose $\Phi_{vv}(s)$ is factored such that

$$\Phi_{vv}(s) = G(-s)\ G(s)$$

with the poles and zeros of G(s) all located in the left half of the complex plane.

Also, let G(s) be separated into two terms

$$G(s) = S(s) + N(s) \tag{2.20}$$

where S(s) contains all the signal poles and N(s) contains all the noise poles, that is, the positive poles of the signal and noise spectra. The random process v could be considered to be constructed from white noise w as shown in Fig. 2.5.

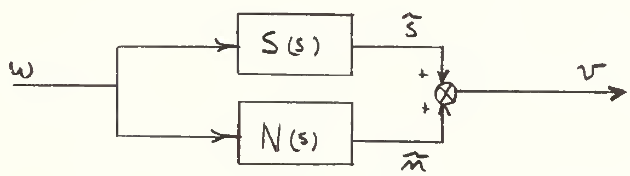


Fig. 2.5 Effective generation of noisy input random process

Ref. 1 proves that the optimum linear operation on the random process v merely reproduces the output $\tilde{s}$ of Fig. 2.5 as the best estimate of the actual signal s. This optimum system is shown as a simple unity feedback configuration in Fig. 2.6. This representation is also valid when signal and noise are correlated.

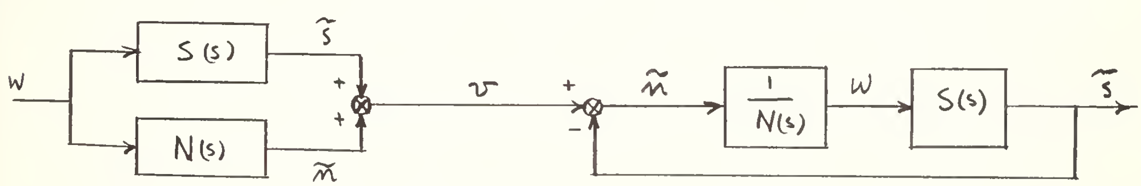


Fig. 2.6. Simple feedback configuration for optimum filtering

This is the first example of the utility of postulating a model for creation of the random process, contrasting with the laborious mathematical treatment found in standard texts.

b. The optimum predictor problem

It is desired to produce continuously the best estimate of a future value of a random process in a mean square sense. This problem (more specifically the prediction of aircraft motion for fire control applications) was the motivation for Wiener's wartime work[22] which laid the foundation for modern random process theory.

If one postulates a model G(s) for the process v, where

$$G(-s)\ G(s) = \Phi_{vv}(s)$$

then this transfer function model is characterized by a state just as any linear dynamic system is. These state variables are analogous to a sufficient set of initial conditions needed to determine the future output at v if the white noise excitation were suddenly cut off.

The optimum predictor, as proved in Ref. 1, is one which continuously produces or recovers the state variables of the model G(s) and weights each by its value after τ seconds of initial condition decay. What this means is that the best guess of the future value of a random process is made by neglecting any future effect of white noise and only by considering the future action of present signal levels in the assumed model G(s), which summarize the total effect of the past theory of the random process as it can affect the future.

The importance of this result transcends the specific application of a predictor. Since the expected value of future signal in a random process is found by allowing the random process state variables to decay as initial conditions, these quantities assume prime significance in the design of a control system to operate in the random environment. These matters will be considered in some detail in Chapter 4.

2.4 Multi-dimensional random processes

By far the majority of theoretical investigations into random processes and the optimum systems which operate on them have been concerned with single or scalar signals. When two or, in general, n random processes are to be processed simultaneously, the theoretical aids to understanding have been essentially missing. One of the major purposes of Ref. 1 was to meet this need, especially motivated by the desire to examine random behavior of ships at sea. This section will outline the problem and a general theory for multi-dimensional random processes.

First, it is necessary to consider what a multi-dimensional system is. We will concern ourselves with linear systems which have inputs and outputs of n signals. Fig. 2.6 shows the structure to be considered. Since each output q_i can be caused by any or all of the inputs v_j, an entirely general representation of W(s) is the definition

$$q_i(s) = \sum_{j=1}^{n} W_{ij}(s)\, v_j(s) \qquad (i = 1, 2 \ldots n)$$

or

$$q(s)\big] = \underline{W(s)} \quad V(s)\big] \tag{2.21}$$

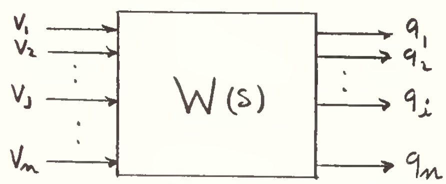
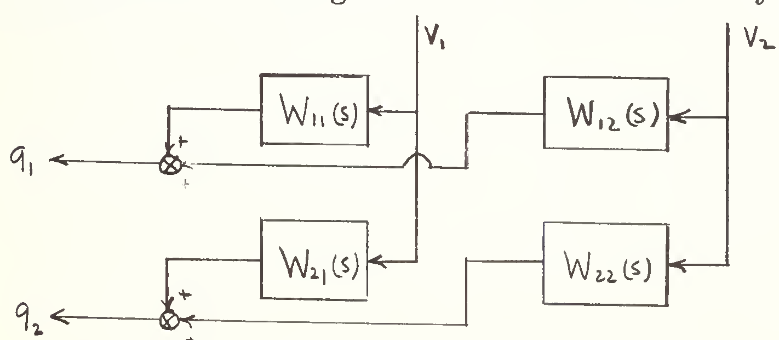


Fig. 2.6 A general multi-dimensional system.

Each element of $\underline{W(s)}$ is a separate transfer function. Fig. 2.7 demonstrates the configuration of a 2x2 matrix system.



$$\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} W_{11}(s) & W_{12}(s) \\ W_{21}(s) & W_{22}(s) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Fig. 2.7 System defined by 2x2 matrix

Next, we consider a single set (or "vector") of signals, v, and inquire as to what information must be known in order to completely specify it as a random process. Obviously, from the previous section, the only measures we can use involve correlation and thus specification of the power density spectrum $\Phi v_i v_j(s)$ for i, j = 1, 2 . . . n is sufficient to define the process statistically for our purposes. In matrix form, an nxn matrix with ijth element $\Phi v_i v_j(s)$, $\underline{\Phi_{vv}(s)}$, represents v. Similarly, the cross-power spectral relation $\Phi v_i q_j(s)$ is the ijth element of a matrix $\underline{\Phi_{vq}(s)}$.

A general formula for the matrix of cross-power density spectra, $\underline{\Phi_{xy}(s)}$, between two vector random processes, x and y, is

$$\underline{\Phi_{xy}(s)} = \sum_{i=1}^{n} \sum_{j=1}^{m} \underline{G_i(-s)}\ \underline{\Phi_{x_i x_j}(s)}\ \underline{H_j^T(s)} \tag{2.22}$$

where

$$\left.X(s)\right] = \sum_{i=1}^{n} \underline{G_i(s)} \ \left.X_i(s)\right]$$

and

$$\left.Y(s)\right] = \sum_{j=1}^{m} \underline{H_j(s)} \ \left.Y_j(s)\right]$$

The superscript $[\ ]^T$ indicates the matrix transpose. This simple formula is analogous to the scalar case, Eq. 2.19.

To illustrate a simple application of this result, consider again the system of Fig. 2.6. Here,

$$\underline{\Phi_{qq}(s)} = \underline{W(-s)}\ \underline{\Phi_{vv}(s)}\ \underline{W^T(s)} \tag{2.23}$$

and

$$\underline{\Phi_{vq}(s)} = \underline{\Phi_{vv}(s)}\ \underline{W^T(s)} \tag{2.24}$$

Suppose we next inquire as to the output relations of a white noise driven system, since this concept supplied so much insight into the scalar problem. Here, the white noise is an n-dimensional vector of uncorrelated white noise with unit power density spectra. $\underline{\Phi_{ww}(s)}$ in this case is $\underline{I}$, the identity matrix. The output vector v of a matrix system $\underline{G(s)}$ is, from Eq. 2.23,

$$\underline{\Phi_{vv}(s)} = \underline{G(-s)}\ \underline{G^T(s)} \tag{2.25}$$

Let us now ask the reciprocal question --if $\underline{\Phi_{vv}(s)}$ is found by measurements on v, can a $\underline{G(s)}$ be determined which satisfies the above relation and could create the process from white noise? This is a problem of matrix factorization and, once solved, it serves to unlock the door to the understanding of multi-dimensional random processes.

A general method is presented in Chapter 3 of Ref. 1 which always leads to a solution to this problem. We will not concern ourselves here with the details, save to say that the procedure used is valid without regard to order and consists of elemental operations such as polynomial factoring and solutions of linear equations. However, the significance and utility of the answer is of considerable importance.

The solutions to the multi-dimensional filtering and prediction problems are identical to those cited in the last section for the scalar case --if matrices replace the single-dimensional system expressions. Especially important is the fact that the optimum prediction of future values of the multi-dimensional random process is the decay of state variables determined in the model G(s).

2.5 Significance of multi-dimensional theory to ship motion study

A foundation --admittedly sketchy --has now been laid for the presentation of a new approach to statistical understanding of the sea and ship motion. The heart of this approach is the visualization of a random process, single or multi-dimensional, as the output of some linear system which is driven by white noise. The randomness of the random process is caused by the white noise, and the memory or useful part is caused by the linear system.

First, let us consider the sea itself. It is manifestly impractical to monitor a large area of ocean for amplitude disturbance, Project SWOP[16] notwithstanding. However, a finite set of measuring instruments such as wave poles is possible. The signals from these devices comprise a multi-

dimensional random process and can be analyzed by cross-power spectra techniques to produce the matrix of spectra, $\underline{\Phi_{vv}}(s)$. The $\underline{G(s)}$ which satisfies the equation

$$\underline{G(-s)}\ \underline{G^T(s)} = \underline{\Phi_{vv}(s)} \tag{2.25}$$

is the effective system which carries in its signal levels all that is knowable for prediction.

White noise can be visualized as a random sequence of uncorrelated impulses. In the case of the sea, the white noise is physically approximated by the gusts or random wind forces which continually supply energy to the sea. The sea is then a linear system which converts the effect of a single gust into a small displacement disturbance which propagates at the interface and interacts with the contributions from millions of other gusts. This very gross description is intended to make the point that white noise is not necessarily an abstraction, and is actually a major factor in the actual randomness observed at sea.

If we were to use an arbitrarily large number of wave poles, it is possible that we could reduce the prediction error for future wave motion at a given point to less than any desired amount, since the inclusion of an additional signal source which is correlated with the signal in question will always reduce the prediction error.

These ideas are interesting but of dubious practical value since it is a rare physical problem which is concerned with instantaneous values of wave height.

A second use of the multi-dimensional theory occurs when ship motion is to be measured. If the six-dimensional motion of the ship, q, is measured, and the input forces are denoted by v, then from Eq. 2.23,

$$\underline{\Phi_{qq}(s)} = \underline{H^{-1}(-s)}\ \underline{\Phi_{vv}(s)}\ \underline{H^{-1,T}(s)}$$

where $\underline{H^{-1}(s)}$ is the ship matrix transfer function as derived in Section 2.2.

Suppose first that $\underline{H}(s)$ is known, from knowledge of the coefficients of the linear differential equations. Then, the input spectra $\underline{\Phi}_{vv}(s)$ is given by

$$\underline{\Phi}_{vv}(s) = \underline{H}(-s)\ \underline{\Phi}_{qq}(s)\ \underline{H}^T(s)$$

An approach which is less interested in the mechanism of ship motion would take the motion statistics, $\underline{\Phi}_{qq}(s)$, and factor this such that

$$\underline{\Phi}_{qq}(s) = \underline{G}(-s)\ \underline{G}^T(s)$$

This system $\underline{G}(s)$, composed of a mixture of force and motion systems, has in its signal levels or state variables the necessary information for the optimum prediction of future motion of the ship. This would be of considerable practical import for motion control devices, and for perhaps more exotic uses such as guided missile launching installations or automatic landing systems on aircraft carriers.

However valid this parti cular approach might be for a certain spectrum, it is felt that the relatively constant transfer function of the ship should be determined and fixed in any design procedure, and attention be paid to the real variable in the problem, the random process of forces on the ship due to wave action.

The ship coefficients are perhaps best determined in the towing tank, but a useful check can be made for some of the major parameters of interest --such as natural frequencies and damping ratios --by analysis of the correlation functions obtained from reduction of motion data taken on the actual ship at sea. Chapter 3 will illustrate an example of this particular use of these statistical measures in motion analysis of the USS GYATT (DDG 1).

The primary emphasis, then, should be first on fixing the parameters which govern the differential equations of motion of the particular ship, and then investigating the statistical random process of the wave forces on the ship. The latter is best done practically by simulating aboard

ship the equations of motion. This is best shown by means of an example:

Suppose that the roll moment is to be found. From Section 2.2,

$$M_K = (K_{\dot p} + I_x)\ddot{\phi} + K_p \dot{\phi} + K_{\dot\phi} \ddot{\phi} + K_{\dot r} \ddot{\psi} + K_r \dot{\psi} + K_{\dot v} \ddot{y}$$

$$+ K_v \dot{y} - M_c$$

where ϕ, ψ, and y are roll, yaw, and sway displacements, respectively. These motions can be measured quite easily with accelerometers, rate gyros, etc., and weighted and summed as shown. The control term M_c refers to the moments generated about the roll axis from, for example, fins or rudders. These forces are probably best introduced into the problem from strain gauges on the rudder or fin stock or other signal sources which give an output proportional to actual lift force obtained, not to the deflection angle of the appendage.

The result of this measuring and weighting for all dimensions would be a set of six random processes recorded simultaneously. The cross-power density spectrum $\Phi_{v_i v_j}(s)$ would be calculated between signals, and the resulting matrix $\underline{\Phi_{vv}(s)}$ factored into $\underline{G(-s)}\ \underline{G^T(s)}$. For the purposes of the ship, the sea is mathematically represented over the interval of data taking as a six-dimensional matrix filter $\underline{G(s)}$ excited by six independent white noise sources.

A library of these $\underline{G(s)}$ representations could be established. Typical patterns could be determined and useful approximations could be made in their configurations as various sea conditions were "tested" by the ship. Hopefully, the sea could be well-characterized by a few parameters of the matrix.

A very important use of the $\underline{G(s)}$ representation is that it is in a form suitable for direct instrumentation on an analog computer, using commonly-available white noise sources to excite this linear system. Thus the effects of, for example, a strong correlation between pitch and heave excitation could be experimentally analyzed for the purposes of control.

It is a reasonable assumption that $\underline{G(s)}$ representation of random seas would not differ greatly for hull forms of ordinary proportions in the same seaway. Thus, if a typical $\underline{G(s)}$ were specified after some experience with this statistical measure of the sea, the effect of coefficient changes of a particular ship could be evaluated through considering the matrix

$$\underline{\Phi_{qq}(s)} = \underline{H^{-1}(-s)} \; \underline{G(-s)} \; \underline{G^{T}(s)} \; \underline{H^{-1,T}(s)} \qquad (2.23)$$

The diagonal elements of $\underline{\Phi_{qq}(s)}$ are the auto spectra of roll, pitch, etc. The mean square value of these quantities is the zero value of the corresponding inverse Laplace transform. Thus an analytical relation is available for studying the magnitudes of motion of a ship in a standard sea as a function of the coefficients in its differential equations.

With the increasing use of digital computers for complex problems in naval architecture, it seems possible that quite an ambitious scheme for hull design could be formulated. From a set of equations for hull forms with several variables, the coefficients of the differential equations could be calculated by strip theory. These coefficients could be used to evaluate mean-square motion of the ship to the standard seaway $\underline{G(s)}$, and the result used inside a loop in the computer to find the optimum set of hull parameters.

The most important immediate use of this multi-dimensional random process appraoch is in control. This interesting problem will be dealt with in more detail in Chapter 4.

CHAPTER III.

PRACTICAL PROCESSING OF RANDOM PROCESS STATISTICS

3.1 Introduction

In this chapter we will be concerned with the problem of obtaining useful information from actual measurements made on random processes in the form of correlation functions or their transform mates, the power density spectra. The theoretical results of this thesis, and indeed all basic random process theory, depend upon the ability to approximate correlation functions by exponentials or damped sinusoids, or alternately, to represent spectra as meromorphic functions of s, that is, with numerator and denominator which are polynomial functions of s. This does not necessarily pose a significant restriction on the accuracy of the theory, however. Laning and Battin[19], for example, have shown that correlation functions can be approximated by the exponential class of functions to within any desired accuracy level.

In advocating this approach, the author hopes to supplant the well-known Neumann spectrum, presented in Chapter 1, with a less rigid model for the presentation of wave-height statistics. In fact, the term $e^{\frac{K}{s^2}}$, which Neumann used in approximation of early experimental spectra, is analytically untractable for the purposes of random process theory. The alternate results of Voznessensky and Firsoff[12] however fit in directly with this approach, using an exponentially damped cosine wave to approximate many measured wave-height auto-correlation functions.

Also, in several examples of correlation functions of ship motion to be presented in this chapter, the exponentially-damped sinusoid will be seen as a fundamental building block. These particular statistics are representative of measurements made aboard the USS GYATT (DDG 1) by Marks and Durcovic[8] in early 1959.

3.2 Correlation function analysis

In this section, methods will be examined for approximating a correlation function with the following terms

$$\varphi(\tau) = \sum_i C_i e^{-a_i \tau} \cos(\beta_i \tau + \phi_i)$$

allowing the special case with β_i and ϕ_i both zero to represent the pure exponential. Additional but less frequent allowable terms are of the form $\tau^n e^{-\alpha_j \tau} \cos(\beta_j \tau + \phi_j)$. This emphasis on decaying sinusoidal functions is made because it corresponds to the usual observed behavior of statistical measures of ship motion and wave-height records.

The standard form for the transfer function of an under-damped second order system is $\dfrac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2}$. ζ is defined as the damping ratio, and ω_n the natural or undamped frequency. A typical transient of this system will contain the term $e^{-\alpha t}\cos\beta t$. a and β are related to the standard parameters, ζ and ω_n, as follows:

$$a^2 + \beta^2 = \omega_n^2 \quad \text{and} \quad \frac{a}{\omega_n} = \zeta$$

These relations are best visualized with the aid of relations shown on a plot of the system poles on the complex plane, as shown in Fig. 3.1.

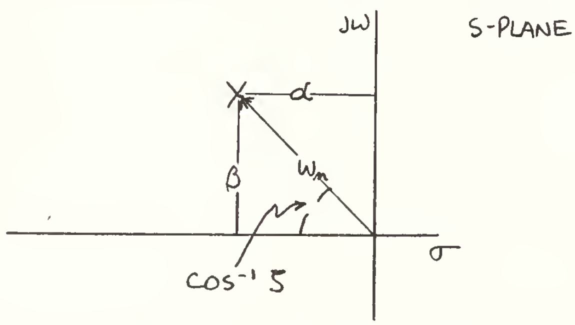


Fig. 3.1 s-plot of standard second-order pole

These standard parameters have been adopted because of their utility in plotting frequency response in control systems applications. The damping ratio governs the rate of decay of amplitude of oscillation. Fig. 3.2 shows the amplitude ratio of successive positive and negative peaks as a

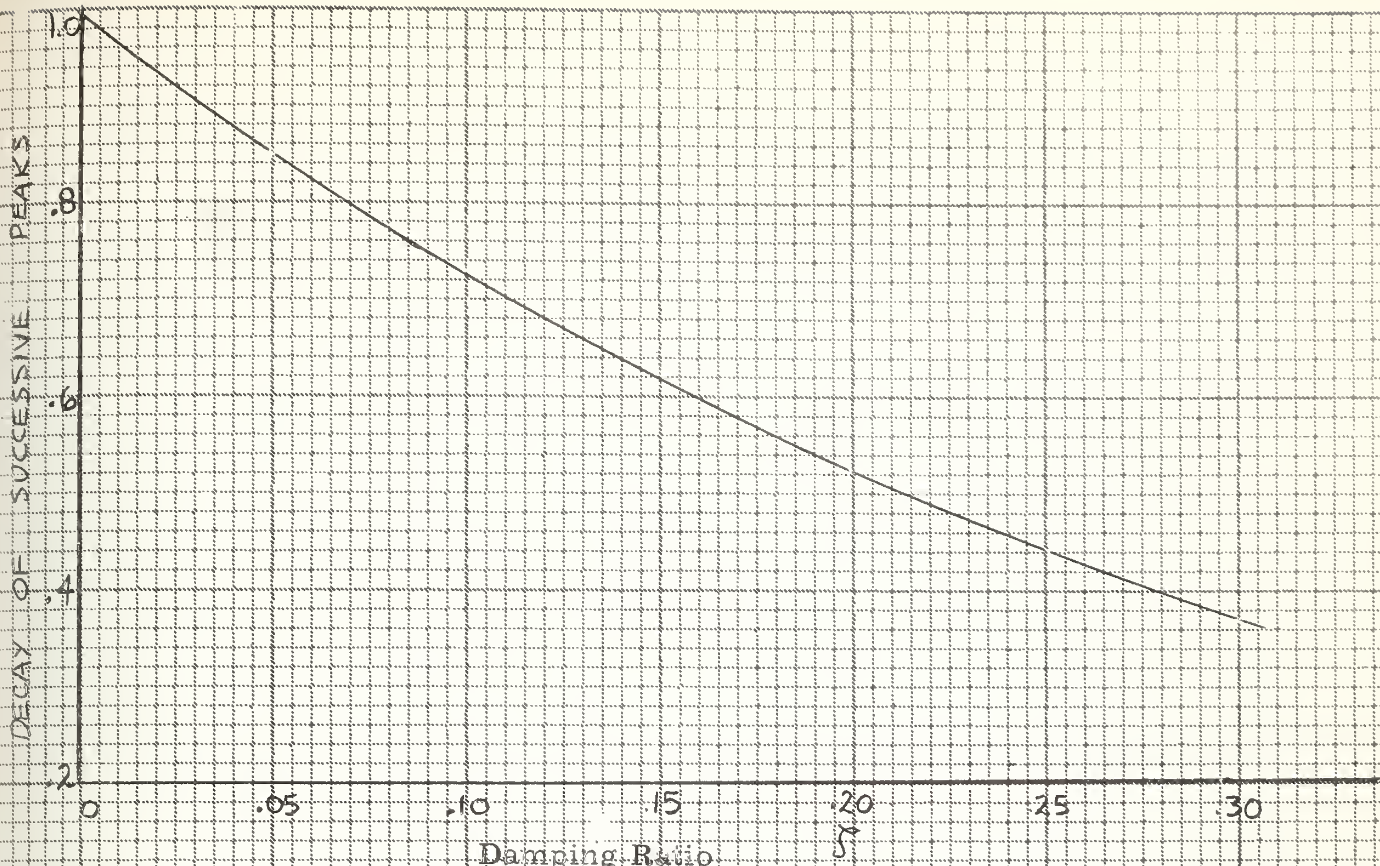


Fig. 3.2 Oscillatory decay as a function of damping ratio

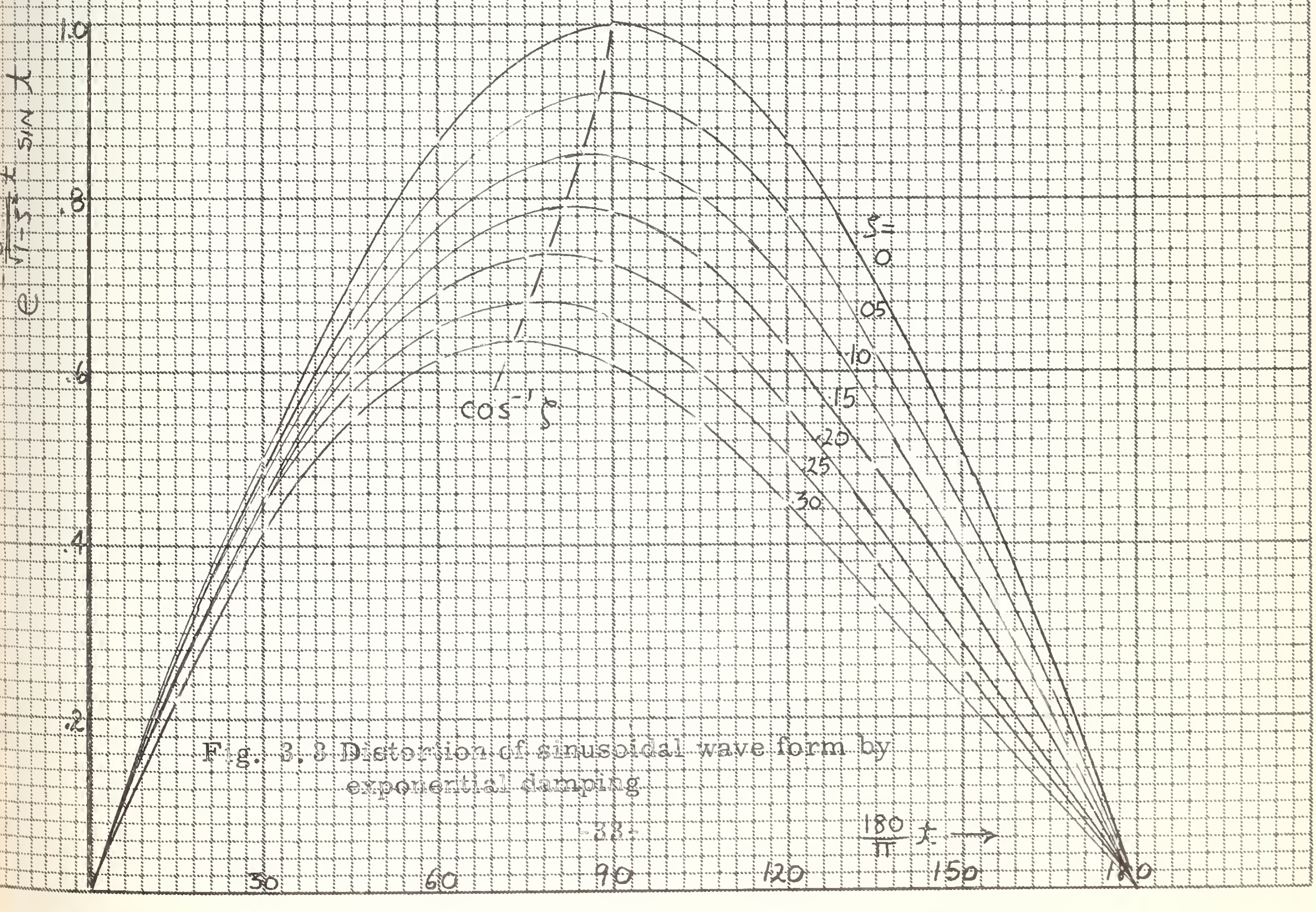


Fig. 3.3 Distortion of sinusoidal wave form by exponential damping

function of ζ for the useful ranges of damping in ship motion.

Another characteristic of the damped sinusoid, the distortion of the normal sinusoidal waveform because of exponential attenuation, is presented in Fig. 3.3.

When it is desired to approximate a correlation function which has been experimentally determined by one or more damped cosine waves, the procedure becomes quite difficult if two nearly equal frequencies are involved, because of a "beat" phenomenon. In this case, the spectral or transform methods to be described in the next section become very attractive. In the remainder of this section, some simple aids will be given to speed determination of the characteristics of single significant sinusoidal components of correlation functions.

The first example to be considered is an auto-correlation function $\varphi_{rr}(\tau)$ of the roll angle of the USS GYATT (DDG 1), measured in state five and six seas[8]. Computed values at one-second intervals are shown in Fig. 3.4, for positive values of τ.

First, the approximate times of peaks are determined and averaged, yielding 4.46 seconds as the mean time between adjacent positive and negative peak values. Since $.46\,\beta = \pi$, then $\beta = .705$.

The interpolated absolute magnitudes of $\varphi_{rr}(\tau)$ at these peak times are plotted on semi-log paper. Using the peak values eliminates the effect of the sinusoidal term; using a log presentation allows an exponential to plot as a straight line. Figure 3.5 shows these points and a superimposed straight-line approximation. The intersection corresponding to the time constant of this line and the $\frac{1}{e}$ ordinate is at time 17.1 seconds. Thus $\alpha = \frac{1}{17.1} = .0583$. The standard parameters are then $\zeta = .082$ s and $\omega_n = .707$.

The approximation $e^{-.0583\tau}\cos .705\tau$ is plotted in Fig. 3.6, and shows quite a good agreement with the measured values. It is heartening

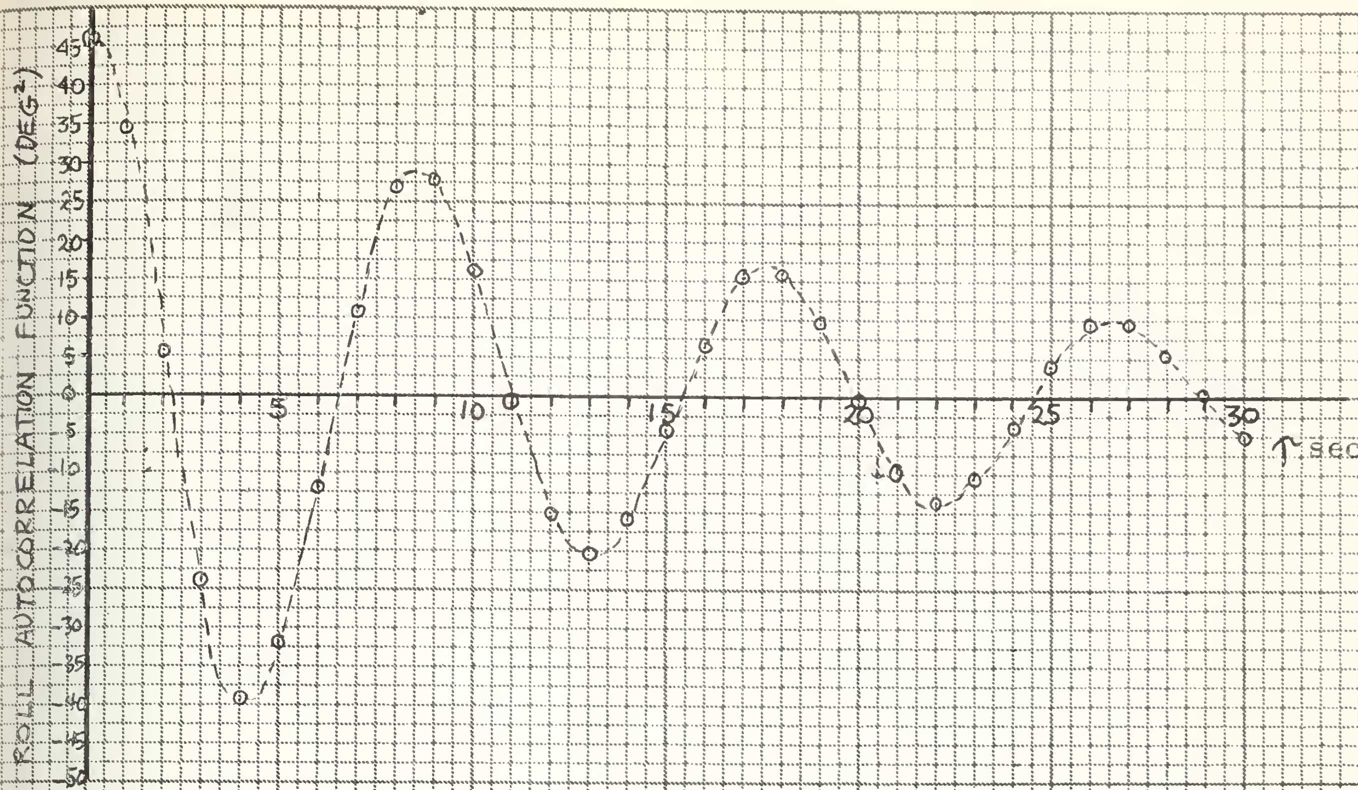


Fig. 3.4 Measured roll autocorrelation function of USS GYATT (DDG 1)

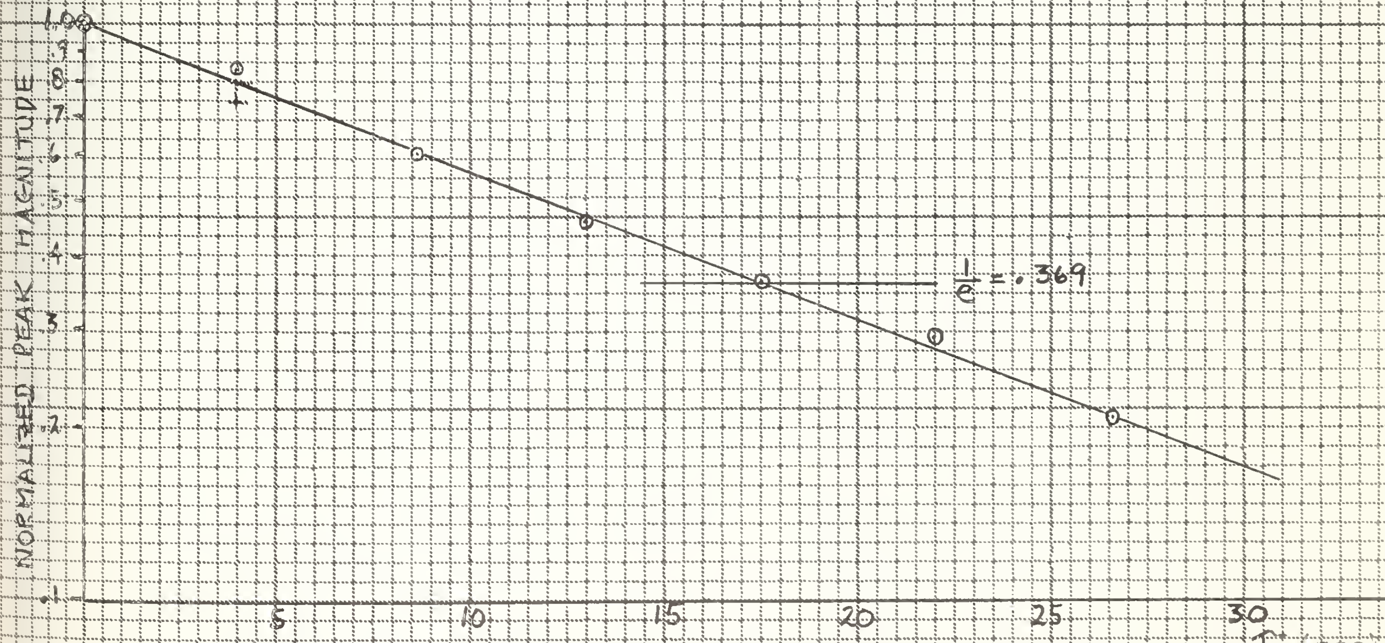


Fig. 3.5 Plot for determining damping of autocorrelation function of Fig. 3.4.

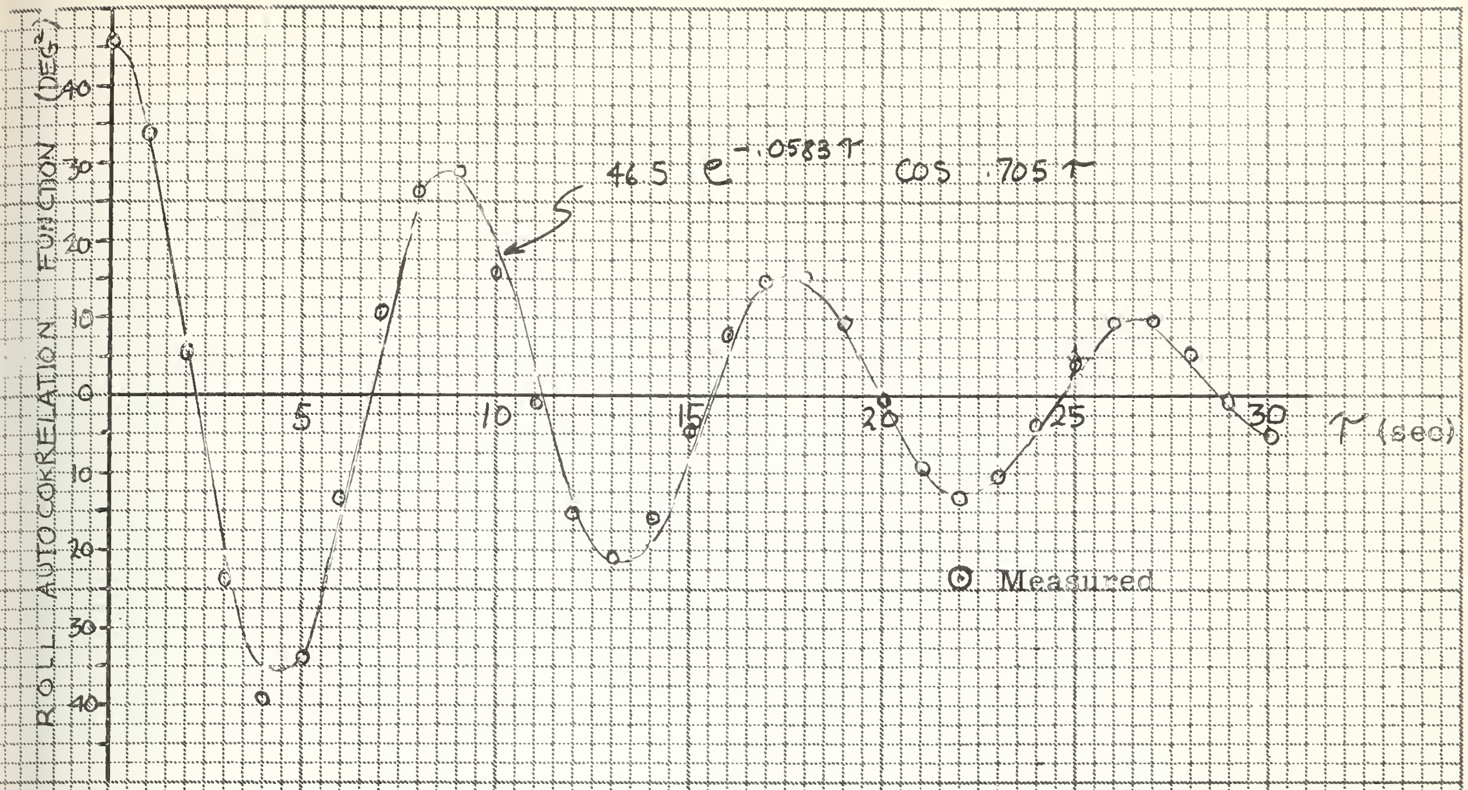


Fig. 3.6 Damped cosine approximation to roll autocorrelation function of USS GYATT (DDG 1)

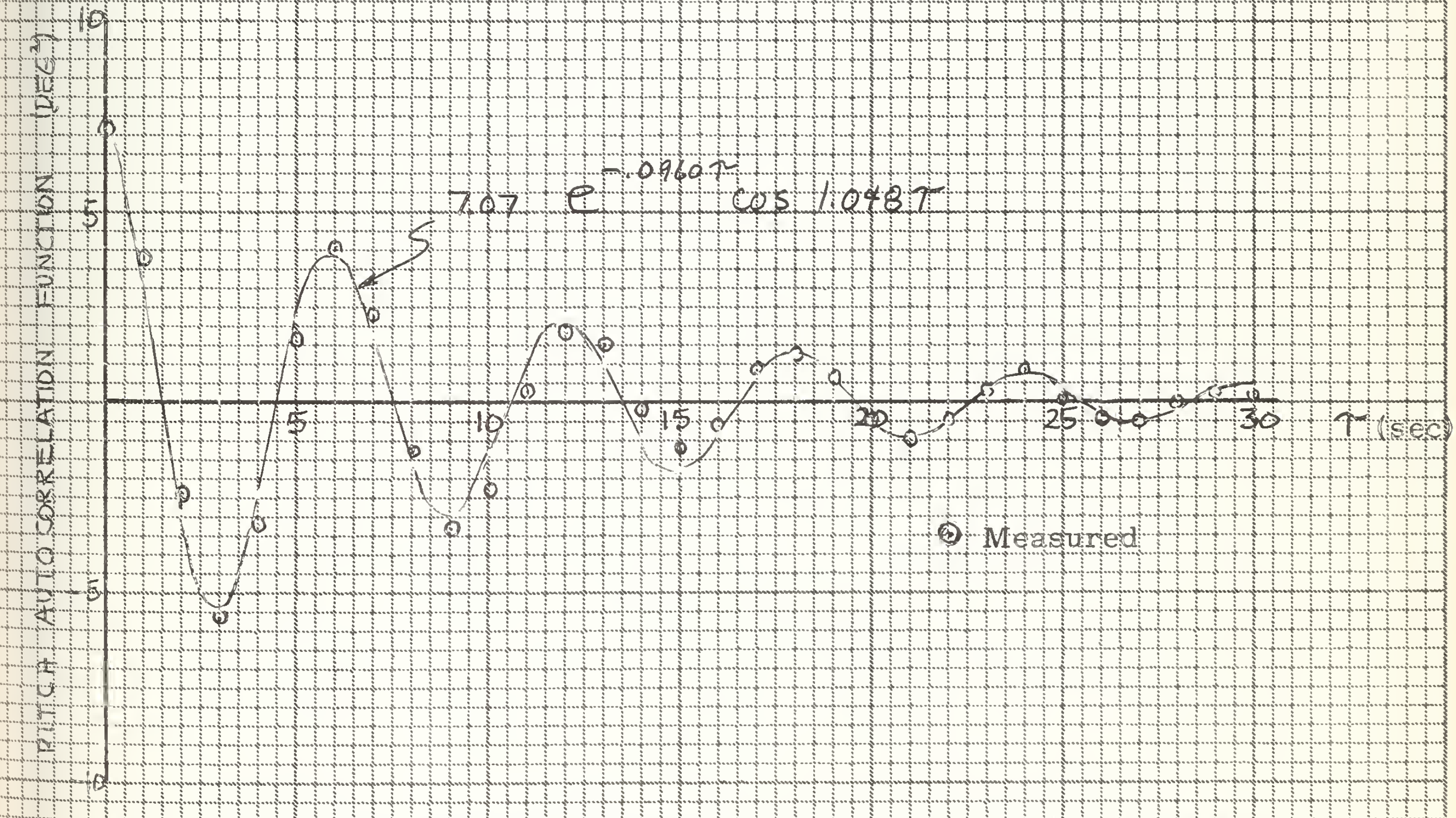


Fig. 3.7 Damped cosine approximation to pitch autocorrelation function of USS GYATT (DDG 1)

to note that this auto-correlation function, which in this case is essentially a measure of the dynamic behavior of the ship in roll to a totally random excitation, corresponds to a linear theory even at large amplitudes of motion at sea. The peak amplitude of roll observed at the time these measurements were taken was 28.5 degrees, which is certainly out of the range of a small-signal theory.

Fig. 3.7 presents a similar approximation to a pitch auto-correlation function, $\varphi_{pp}(\tau)$. The observed characteristics are $\zeta = .0869$ and $\omega_n = .052$. The approximation is not as good as was that of the roll function, which reflects (1) the non-linear aspects of the pitch system at large amplitudes and (2) an indication of the wave excitation statistics.

Figures 3.8 and 3.9 show the measured cross-correlation between roll and pitch. Here $\varphi_{rp}(-\tau) = \varphi_{pr}(\tau)$, a well-known identity . Because of the difficulty in simply expressing these function in terms of damped cosine waves, their analysis is deferred until the next section. However, it is important to note that the cross-correlation measured between these roll and pitch signals does not arise principally because of coupling between the roll and pitch degrees of freedom in the ship system, but rather from the cross-correlation between the exciting moments in roll and pitch from the same wave spectrum.

The significance of correlation function analysis in the time domain is that dominant frequencies and damping ratios can be readily determined, as well as an indication of the non-linearity of the source of the signal.

3.3 Analysis of power density spectra

This section will discuss methods of approximating power density spectra by rational transforms in an efficient fashion. Also, conventional techniques for digital computation of spectra from correlation functions will be critically analyzed.

The heart of this technique is based on methods used by electrical

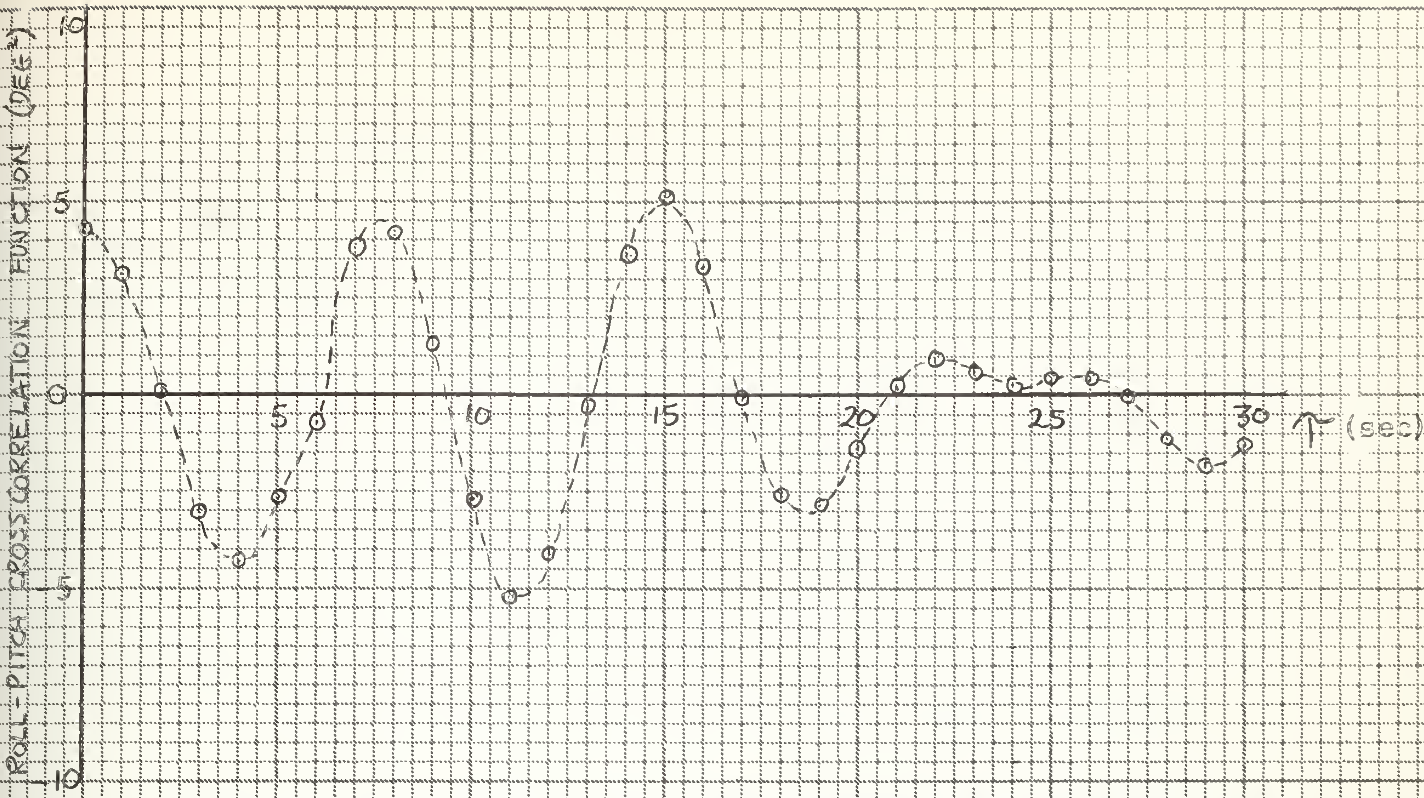


Fig. 3.8 Measured roll-pitch cross correlation function of USS GYATT (DDG 1)

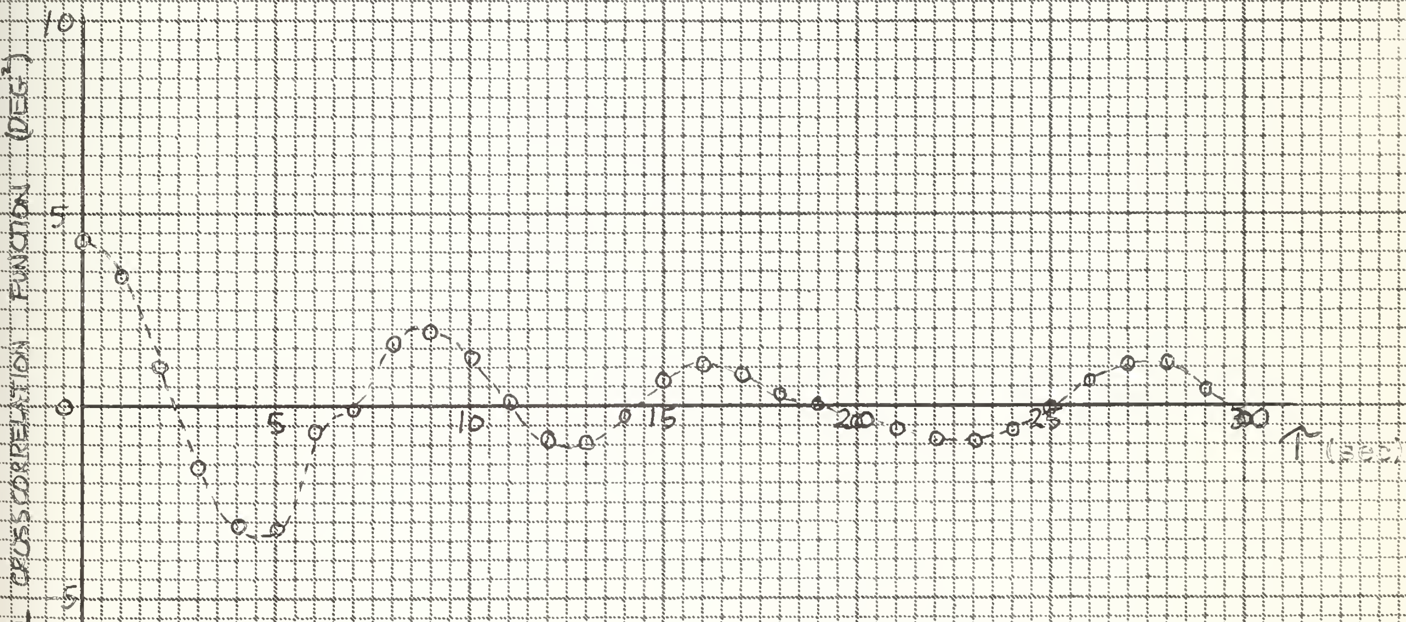


Fig. 3.9 Measured pitch-roll cross correlation function of USS GYATT (DDG 1)

engineers to approximate the frequency response of physical systems with transfer functions in the Laplace variable s. Consider first the auto power density spectrum, $\Phi_{xx}(s)$, which can be represented with the following factors in numerator and denominator:

$$K, \ (\pm s + a_i), \qquad (s^2 \pm 2\zeta_i\omega_i s + \omega_i^2)$$

Suppose the logarithm of $\Phi_{xx}(s)$ is taken. In illustration, if

$$\Phi_{xx}(s) = \frac{K^2(-s+a)\ (s+a)}{(s^2 - 2\zeta\omega_n s + \omega_n^2)(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

$$\log \Phi_{xx}(s) = \log K^2 + \log(-s+a) + \log(s+a)$$
$$-\log(s^2 - 2\zeta\omega_n s + \omega_n^2) - \log(s^2 + 2\zeta\omega_n s + \omega_n^2)$$

In general, each separate pole or zero has an additive effect on the spectra when the latter is expressed in logarithmic form, and thus each can be considered separately.

If s is set equal to $j\omega$, the mirror symmetry of poles and zeros about the imaginary axis in the s-plane for auto power density spectra ensures that each pair contributes a real value only. Thus the magnitude of each pole and zero as a function of ω is the item of interest, and the phase angle is uniformly zero in this application.

For quick graphical approximation, it is customary to plot the frequency function on a logarithmic frequency scale. Fig. 3.10 shows the effect of a single simple pole, $\frac{1}{s+3}$.

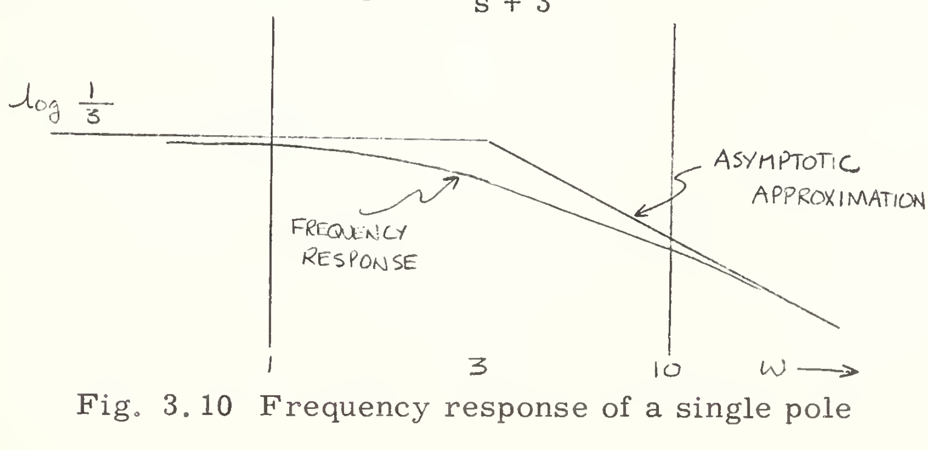


Fig. 3.10 Frequency response of a single pole

Here, the asymptotic approximations reflect the fact that for small ω, $\left|\frac{1}{j\omega + a}\right| \approx \frac{1}{a}$, and for large ω, $\left|\frac{1}{j\omega + a}\right| \approx \frac{1}{\omega}$. Because of the logarithmic frequency scale, the high frequency asymptote plots as a straight line, which makes the entire frequency response of each basic building block shiftable in frequency without distorting its shape. As another example, Fig. 3.11 illustrates the magnitude approximation of a frequency function

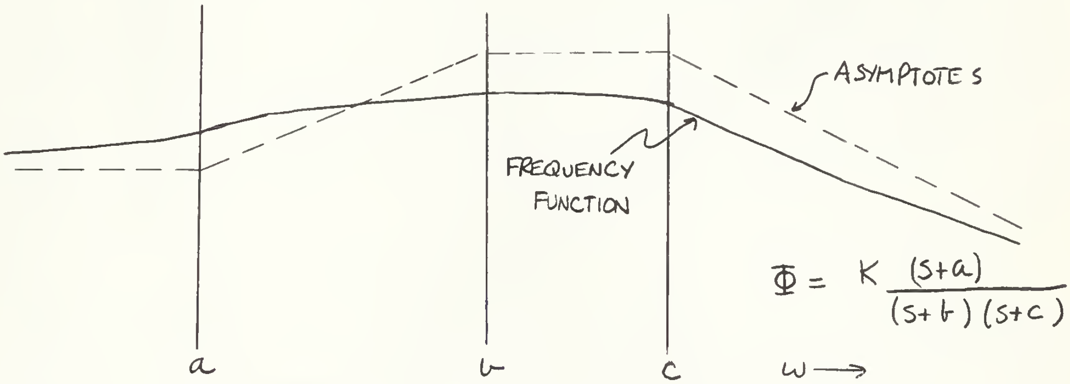


Fig. 3.11. Approximation of frequency function with poles and zeros

with one zero and two poles. Here the asymptotes are plotted all at once, and the positive or negative corrections are added to approximate the observed frequency function. It should be clear that the use of what is effectively a log-log presentation has resulted in considerable simplification in a trial-and-error approximation procedure.

The conventional unit for expressing the magnitude of the frequency function, $F(j\omega)$, is the decibel, defined as $20 \log_{10} |F(j\omega)|$. Although this unit has a meaning of power in audio applications, this restriction is not valid for this frequency response use and the given definition applies to any physical quantity described as a function of frequency. For an asymptote of decay of a single pole, $\frac{1}{\omega}$, the slope must obviously be 20 db fall in a decade of frequency in ω.

Considering the case of auto power density spectra, the log magnitude of each LHP simple pole will be doubled to account for the pair of mirror-symmetric poles. A plot of magnitude of a simple pair of poles of a spectra is given in Fig. 3.12. The reminder is offered that multiplica-

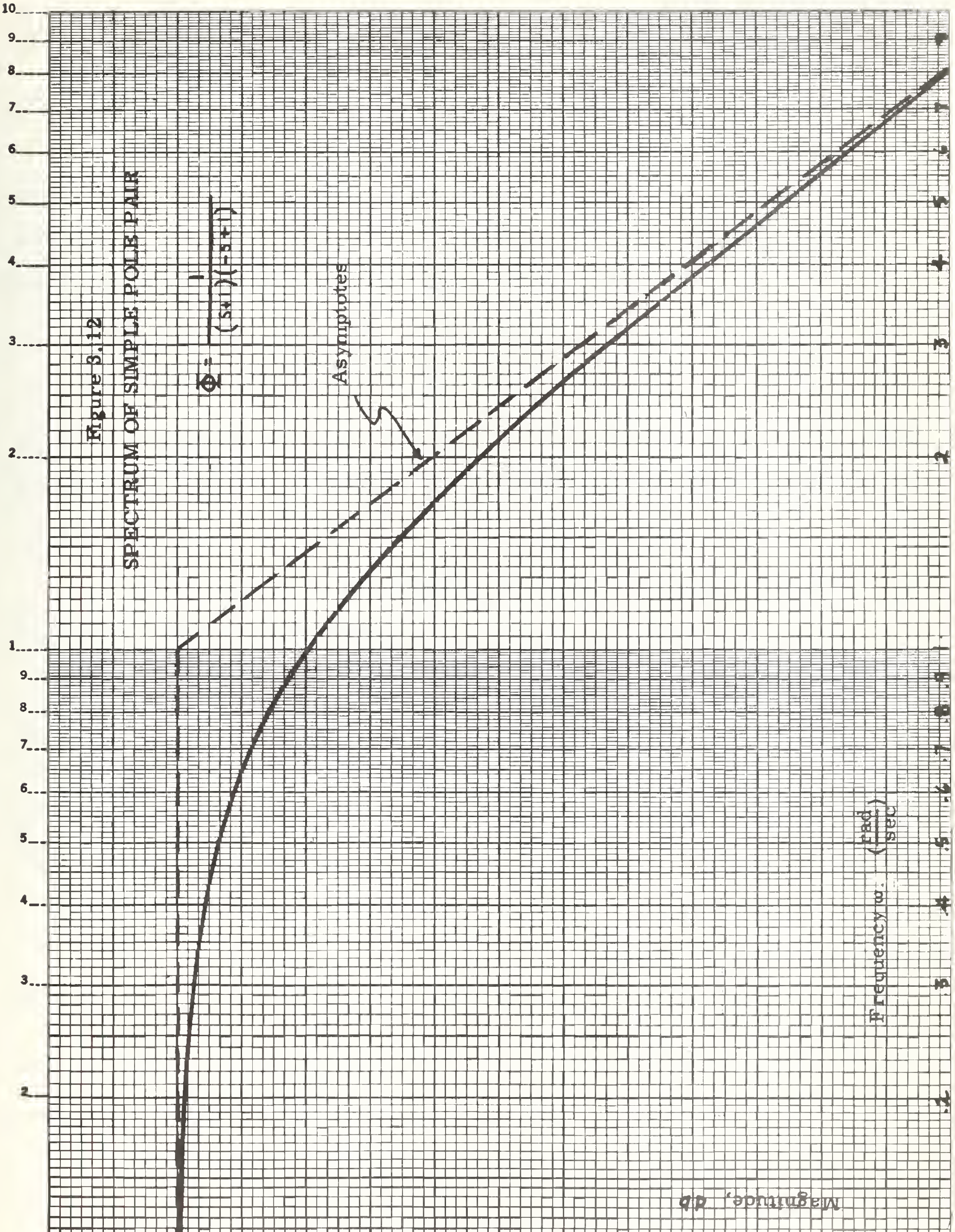


Figure 3.12
SPECTRUM OF SIMPLE POLE PAIR

tion of this spectra by a constant K corresponds to a upward translation of $20 \log_{10} K$ db, and that replacing s by $\frac{s}{a}$ corresponds to a lateral shift of the plot so that the "breakpoint" --or intersection of the asymptotes -- occurs at "a" radians/sec.

Of more immediate interest for the analysis of wave height and ship motion spectra are the poles which are complex-conjugate and indicate sinusoidal behavior of the auto-correlation function. Fig. 3.13 presents the normalized curves for a spectrum of the form

$$\Phi = \frac{1}{\left[\left(\frac{s}{\omega_n}\right)^2 + 2\zeta\frac{s}{\omega_n} + 1\right]\left[\left(-\frac{s}{\omega_n}\right)^2 + 2\zeta\left(-\frac{s}{\omega_n}\right) + 1\right]}$$

over the important range of damping ratios for these applications. Note that the asyptotic slope is now 80 db/decade, indicating the presence of four poles.

If zeros are to be used instead of poles, the same curves can be used with a reversal in sign.

Before using these tools to examine several sample spectra, it is necessary to review means used to determine these particular spectra from their associated correlation functions. The steps to be outlined are given in the authoritative work of Marks[23] and are currently used at the David Taylor Model Basin to produce spectra from digitally computed correlation functions.

A finite duration of auto-correlation function $\varphi_{xx}(\tau)$ is determined at the first M sample points, T seconds apart. Its Laplace transform

$$\int_{-\infty}^{\infty} d\tau\, \varphi_{xx}(\tau) \left\{ e^{-j\omega\tau} = \cos\omega\tau - j\sin\omega\tau \right\} = 2\int_{0}^{\infty} d\tau\, \varphi_{xx}(\tau) \cos\omega\tau$$

is approximated by

$$2\,T \sum_{k=0}^{M-1} \varphi_{xx}(kT) \cos \omega_i kT \qquad (i = 1, 2 \ldots m)$$

This approximation is further smoothed by the operation

$$\Phi_{xx}^{s}(\omega_i) = \frac{1}{4}\Phi_{xx}^{u}(\omega_{i-1}) + \frac{1}{2}\Phi_{xx}^{u}(\omega_i) + \frac{1}{4}\Phi_{xx}^{u}(\omega_{i+1})$$

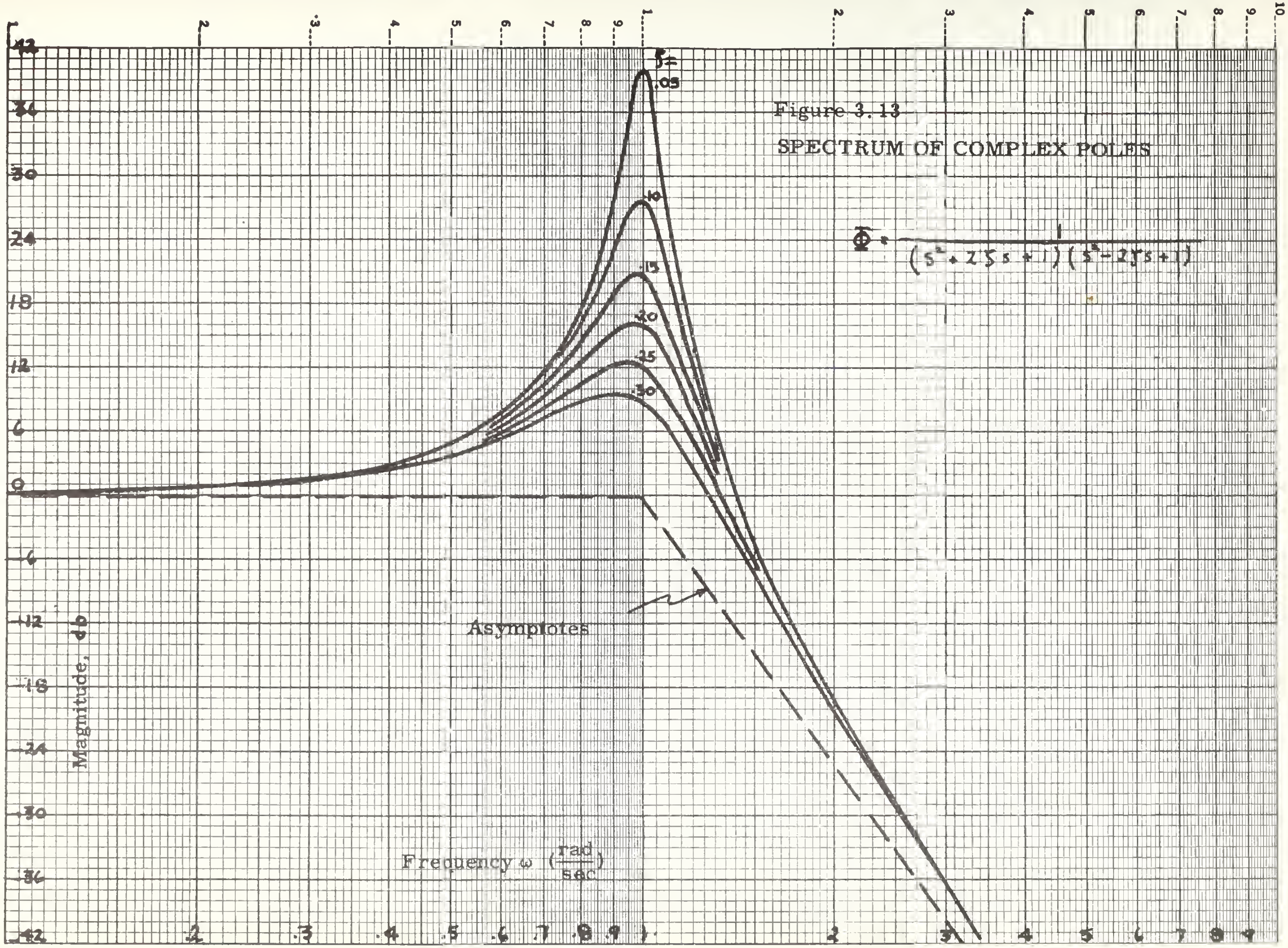


Figure 3.13
SPECTRUM OF COMPLEX POLES

where the superscripts s and u indicate smoothed and unsmoothed respectively.

An auxiliary purpose of the analysis of these spectra will then be to investigate the effects of (1) approximating the infinite correlation function with that of a finite length, (2) approximating a continuous integral by the trapezoidal rule, and (3) smoothing the rough spectra.

The roll auto-correlation function of the USS GYATT (DDG 1), given in Figure 3.6, was excellently approximated by a damped cosine wave, which has a known spectrum. Fig. 3.14 shows the rough and smoothed spectral data computed with the above scheme, and the spectrum of the second-order poles with ω_n and ζ computed in Section 3.2. An excellent fit is obtained near the resonant frequency with the rough computed points of the spectrum. It should be noted at this critical resonant point the smoothed points indicate a greater damping than is actually the case. This is the real penalty for such a crude smoothing operation and easily explained.

Fig. 3.15 indicates two examples of this form of data smoothing. In (a), the smoothed points tend to approach the desired straight line. In (b), a peak in perfect data is considerably flattened because of smoothing.

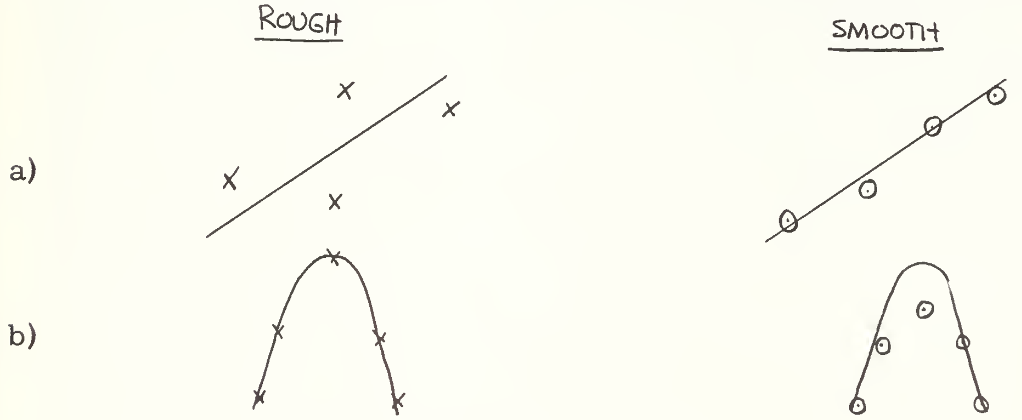


Fig. 3.15 Effects of data smoothing

With the resonant peaks expected in wave height and ship motion spectra,

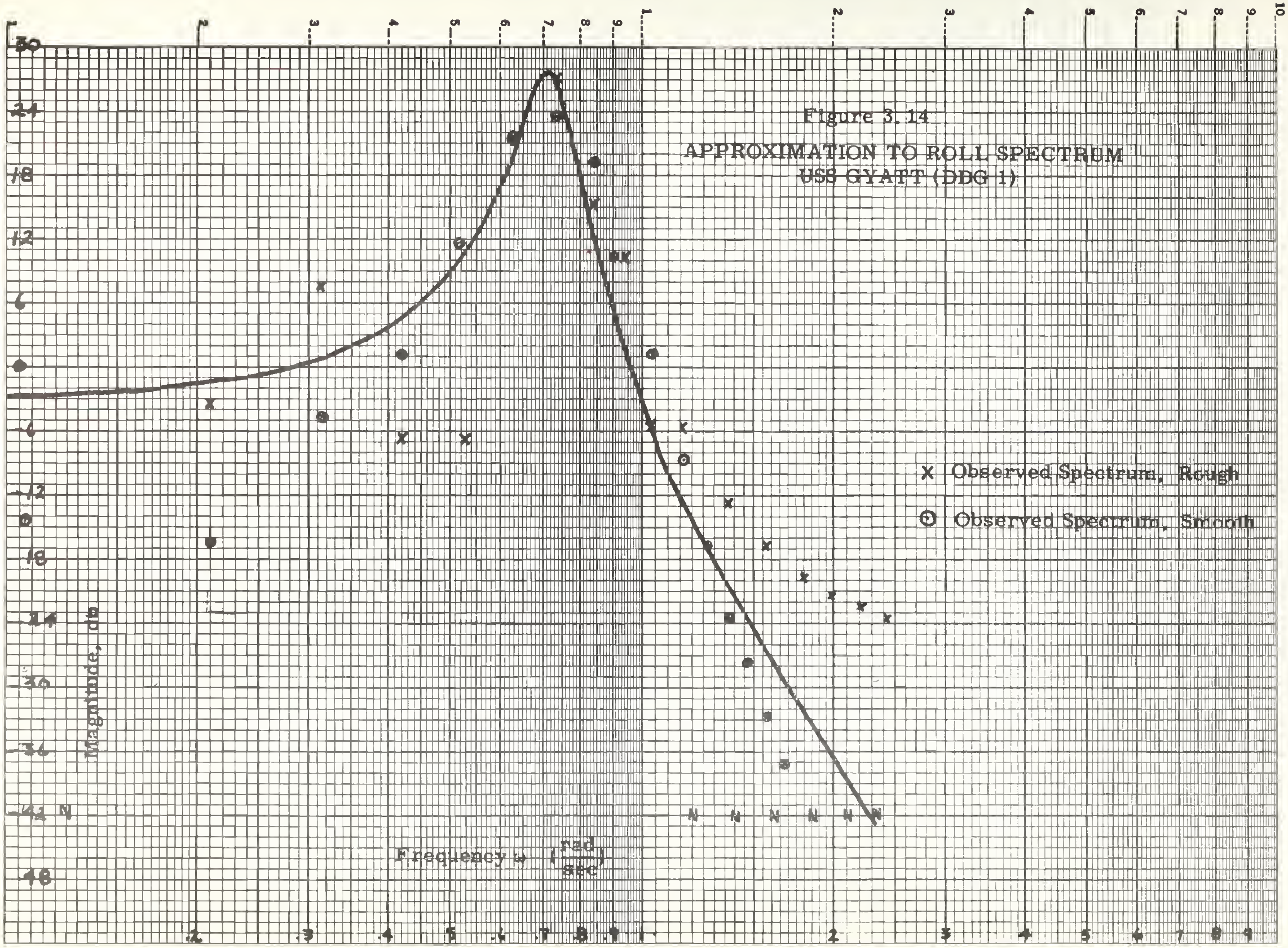


Figure 3.14

APPROXIMATION TO ROLL SPECTRUM
USS GYATT (DDG 1)

the smoothing operation is useful in producing a continuous (if not true) curve away from the resonant point but causes a considerable loss of information about the damping of the system under investigation.

The points in Fig. 3.14 labelled "N" indicate negative power density in the rough spectral computation, which of course, could not occur without imaginary signals. These negative points at low frequencies indicate the effect of a finite (30 second) length of auto-correlation function considered. At high frequencies, the approximation of the integral by a finite summation causes the rough spectral points to be alternately too positive and then negative.

To summarize, the representation of spectra digitally computed by contemporary methods can be considered essentially accurate only in the determination of resonant frequency, less accurate in the determination of damping, and in error by an order of magnitude at two octaves away from the resonant frequency.

Fig. 3.16 shows the rough computed spectra of the pitch signal whose auto-correlation function was given in Fig. 3.7. A good agreement with the exact transform of the approximate damped cosine wave is demonstrated, except for the erratic behavior at low frequencies.

The above examples were fairly straight-forward because only a magnitude function had to be approximated. In the case of a cross-power density spectra, the difficulty is increased by the need to match a phase curve as well as one of magnitude. This is identical to the problem in control systems design, when a transfer function is to be found for a system described by experimentally determined magnitude and phase curves of sinusoidal response.

This procedure is strictly one of intelligent trial and error. Several good references are Chestnut and Mayer[24], who treat the basic problem, and Truxal[20], who discusses more refined estimates in the iterative cycle.

A plot of magnitude and phase for a single pole is given in Fig. 3.17,

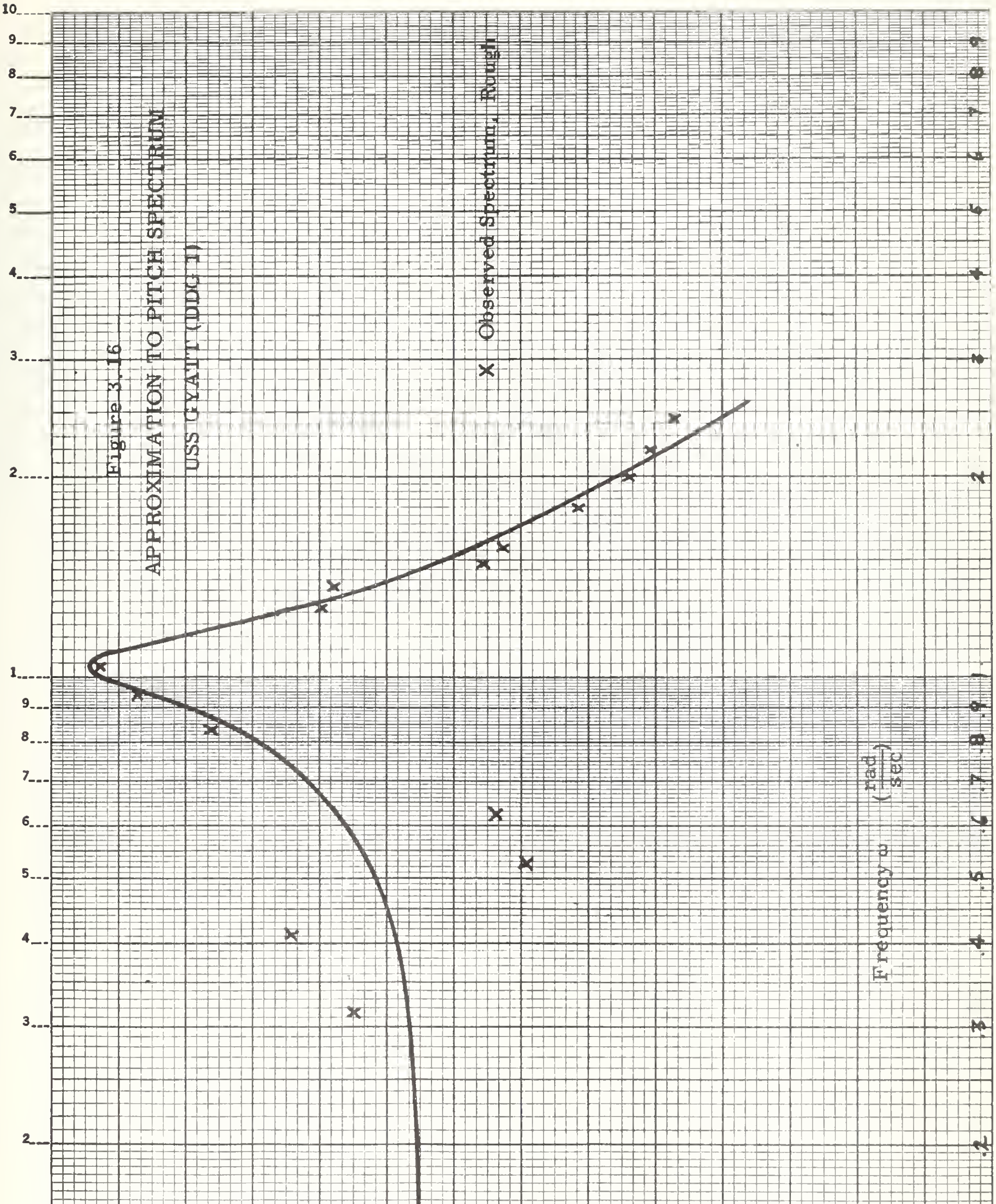


Figure 3.16
APPROXIMATION TO PITCH SPECTRUM
USS GYATT (DDG 1)

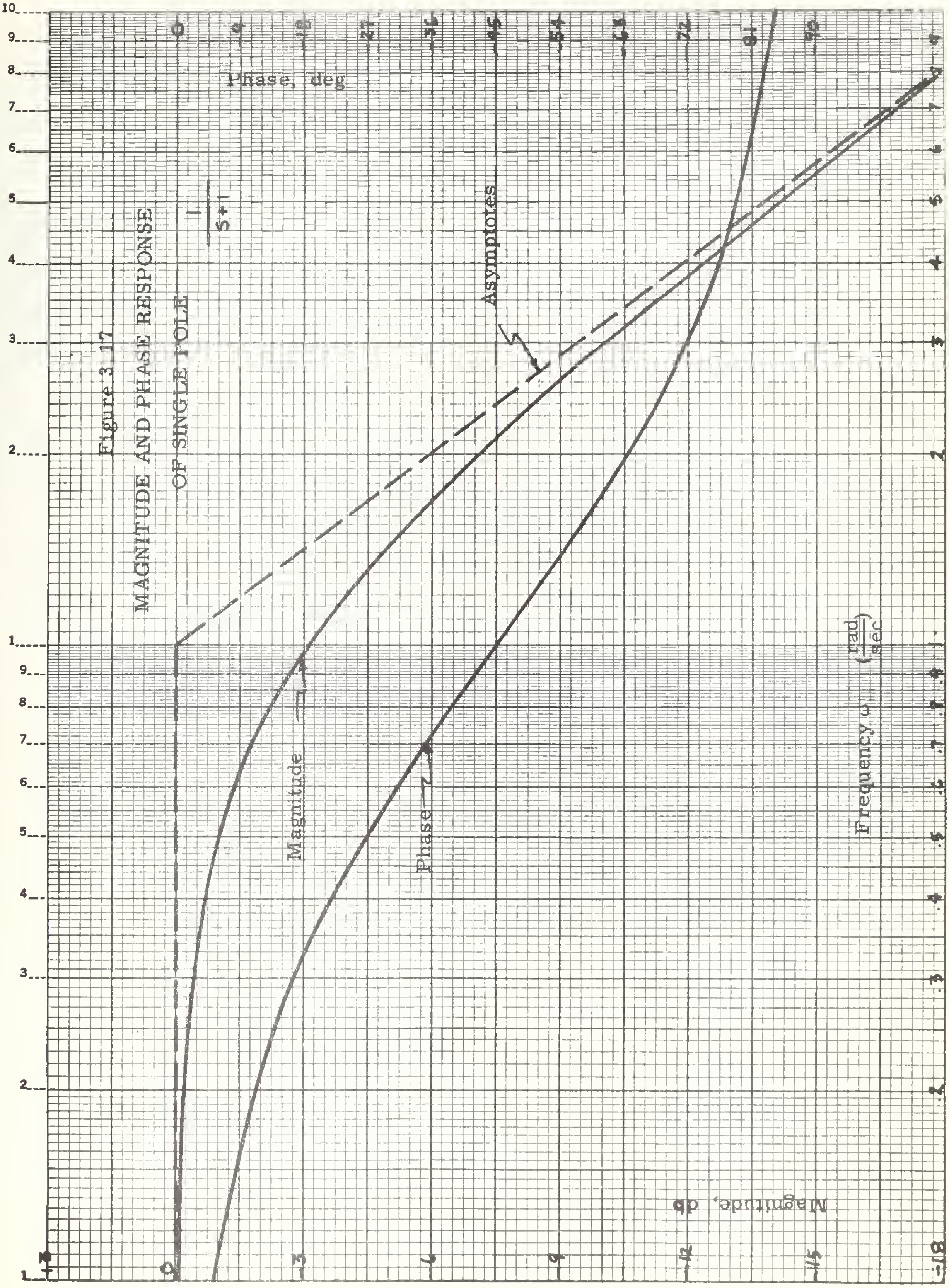


Figure 3.17
MAGNITUDE AND PHASE RESPONSE
OF SINGLE POLE

and for complex poles in Figs. 3.18 and 3.19.

In the case of cross power spectra, one desires to determine the LHP and RHP poles and zeros. The following table lists the correct sign and magnitude of the logarithmic magnitudes and phase angles of Figs. 3.17-19.

	LOG MAGNITUDE	PHASE
RHP ZERO	-	-
LHP POLE	+	+
RHP ZERO	-	+
RHP POLE	+	-

Table 3.1. Sign of magnitude and phase terms

If the cross power spectrum $\Phi_{xy}(s)$ is to be found, it will usually be true that its positive or LHP poles will correspond to those of $\Phi_{yy}(s)$, and its negative or RHP poles will be those of $\Phi_{xx}(s)$.

The roll-pitch cross-correlation functions shown in Figs. 3.8 and 3.9 have a digitally-computed cross spectra given in Fig. 3.20. An approximation over the range of validity appears superimposed. To achieve the proper phase response, it was necessary to introduce a RHP zero and LHP pole pair which were complex and mirror symmetric. Consideration of Table 3.1 indicates that the magnitude effect of these terms is zero, but that the phase effect is additive, giving a net phase shift of -360 degrees at high frequencies.

The analysis of cross-spectra with logarithmic plots of frequency behavior can make no use of the co- and quadrature spectrum records. Accordingly, it is urged that magnitude and phase plots become the standard presentation.

3.4 Conclusions

(1) A review has been made of the normal methods used for the approxima-

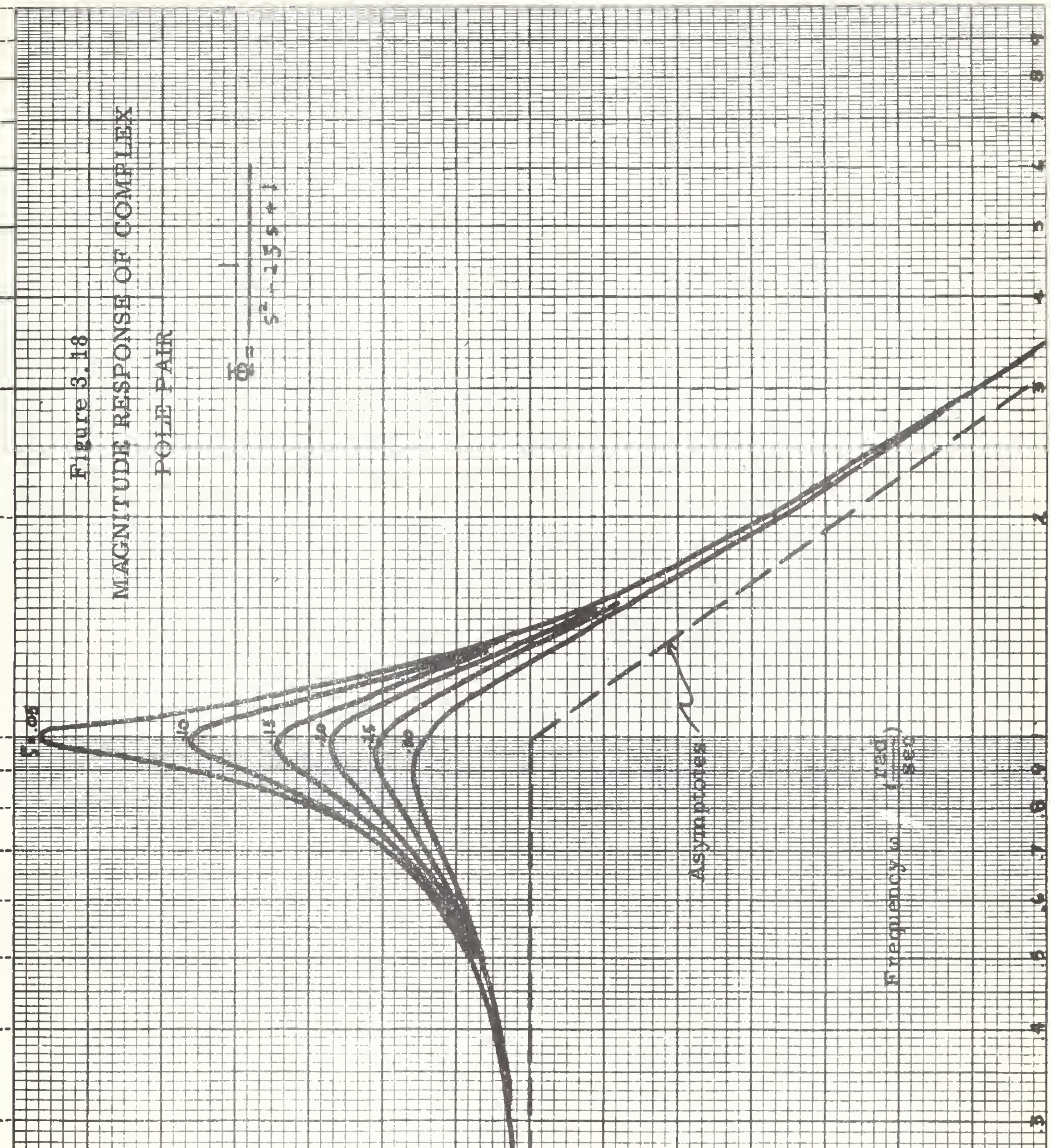


Figure 3.18
MAGNITUDE RESPONSE OF COMPLEX
POLE PAIR
$G = \frac{1}{s^2 + 2\zeta s + 1}$
Asymptotes
Frequency ω (rad/sec)
Magnitude, db
ζ = .05
.10
.15
.20
.25
.30

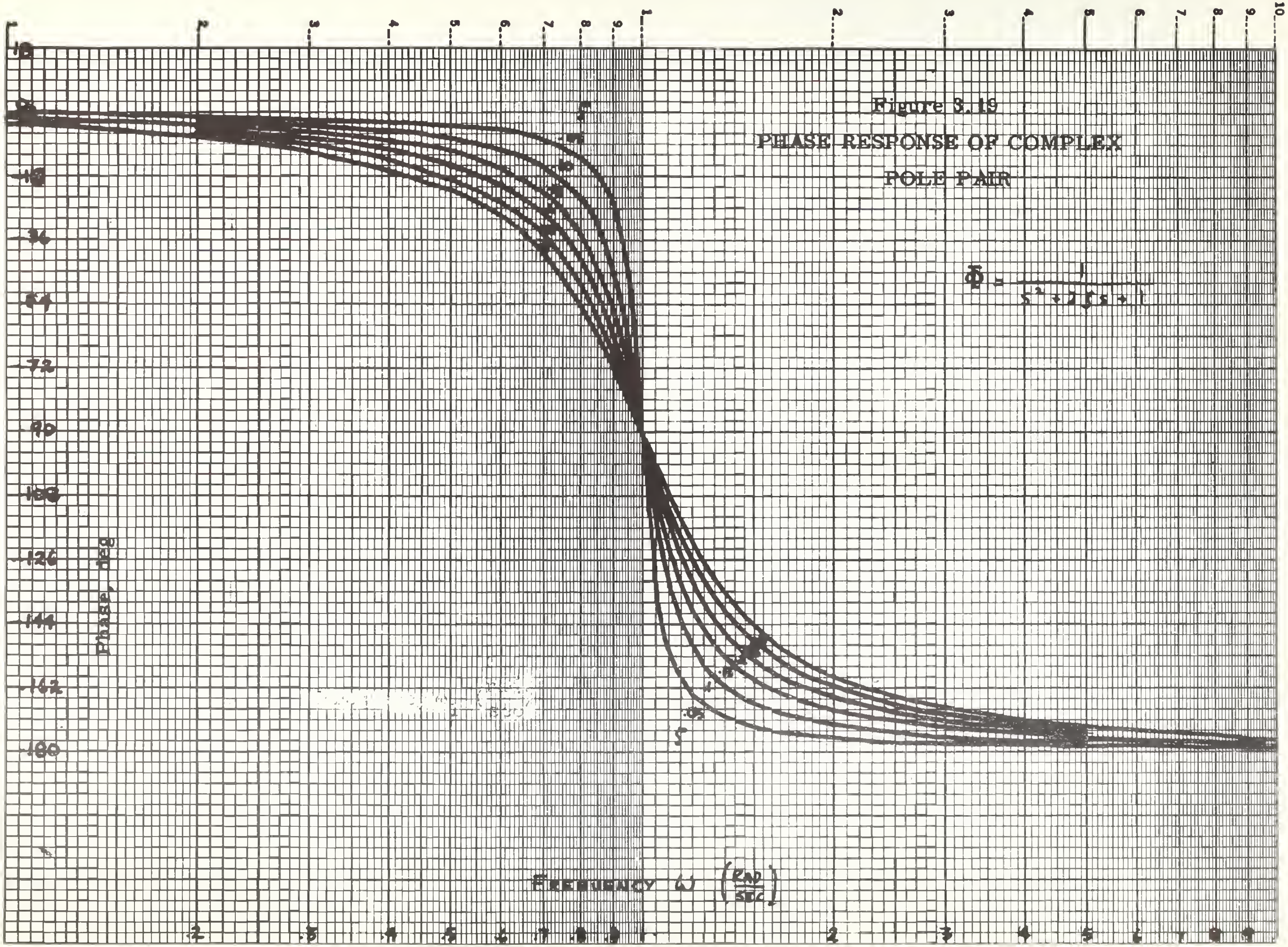


Figure 3.19
PHASE RESPONSE OF COMPLEX
POLE PAIR

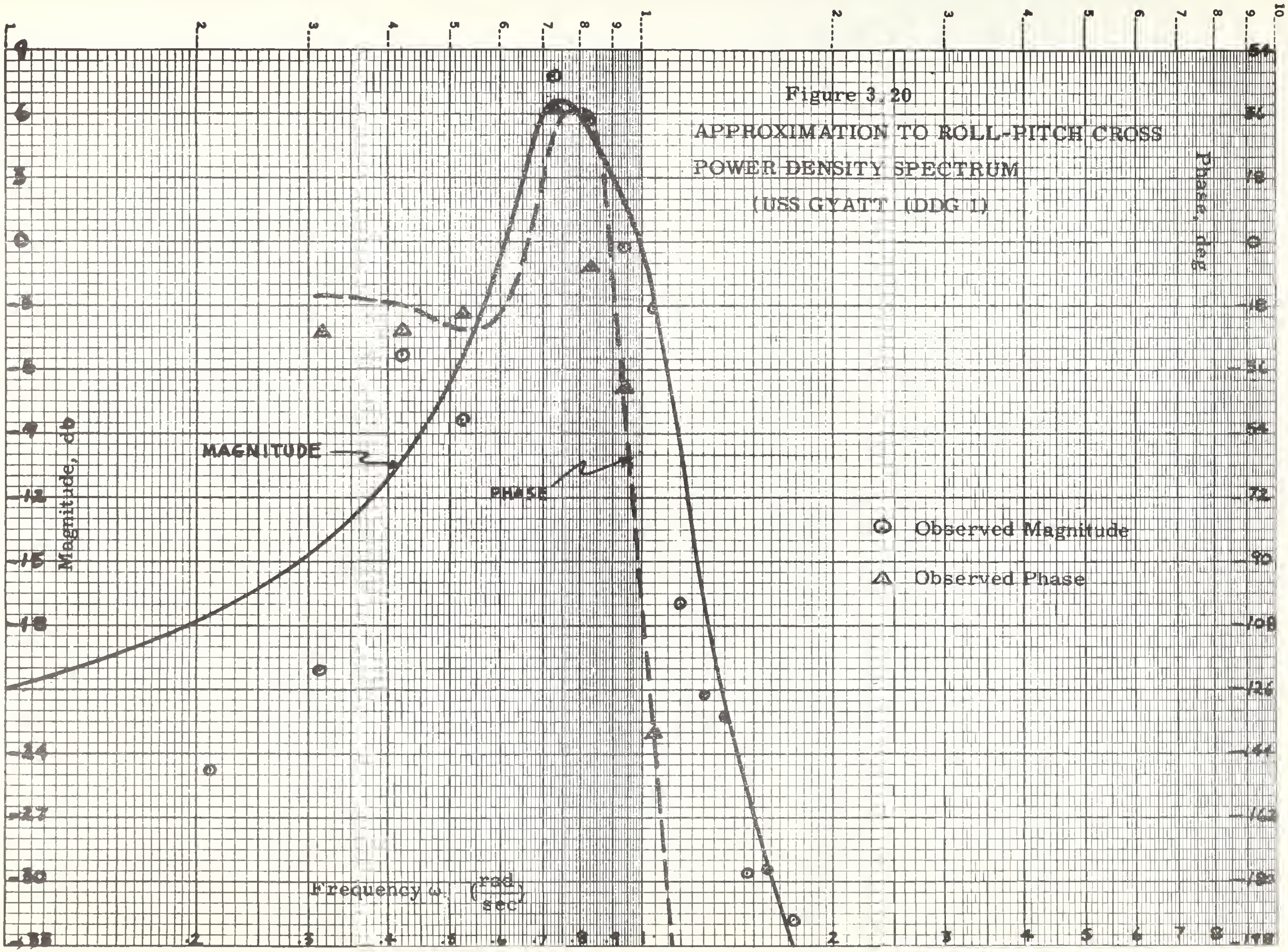


Figure 3.20
APPROXIMATION TO ROLL-PITCH CROSS POWER DENSITY SPECTRUM (USS GYATT (DDG 1)

tion of correlation functions and power spectra density curves with terms usable in a linear theory. No distinct preference has been recorded for either method -- in fact, they supplement each other and in the determination of the dominant frequency and damping values both are equally efficient.

(2) The method currently advocated for the computation of spectra from sampled correlation functions causes considerable error which could be lessened with a more sophisticated computation scheme.

(3) The use of a pole-zero approximation to observed wave-height spectra appears much more desirable, especially if the damping ratio ζ remains sensibly constant in the dominant frequency term[12], than is the preoccupation with the empirically-derived Neumann spectrum.

(4) Statistical analysis of ship motion does not generally reveal useful knowledge of the input wave forces and moments because of the high degree of filtering which the ship system performs on this signal.

CHAPTER IV.

CONTROL OF SHIP MOTION

4.1 Introduction to multi-dimensional ship control

Let us consider a hypothetical ship which is equipped with individually movable fins at both the bow and stern and a rudder. Motion sensors are available to measure any or all of the degrees of freedom of the ship. The design sea is rough, and the peak wave forces exceed the maximum forces from the hydrodynamic appendages because of the latters' finite capacity and breakdown at large angles. It is desired to minimize the average weighted sum of total squared displacement error and total squared velocity of a particular point aboard ship in three dimensional motion, perhaps at a critical missile launching station, by simultaneously controlling each of the five control surfaces. The interactions in the various degrees of freedom of the ship are to be accounted for, as well as the statistical interrelationships of the seaway. In order to maximize the stabilization per dollar expended for the appendage area, these fins and rudders are to be driven full throw.

In short, this problem is one of the most difficult in control engineering. This chapter will present the core of a theory which will allow solution of this and other related stabilization problems in a near-optimum and easily-mechanized fashion. The theory will be applicable to surface ships, submarines, hydrofoil craft, "hydro-skimmers" -- in short, in any marine vehicle problem where the sea exerts the disturbing random force and it is desired to stabilize with several control variables.

4.2 Inadequacy of a linear theory

Chapter 2 has dealt with random processes and the optimum systems to operate on them. It is logical to inquire if there is an optimum linear system for the ship control problem. In a related example, Fig. 4.1 shows

an input random process v, a scalar fixed system H(s), and a controller KC(s) which acts to reduce the output error e. In this simple case it is obvious that including an infinite gain in the controller, (ie: $K \rightarrow \infty$)

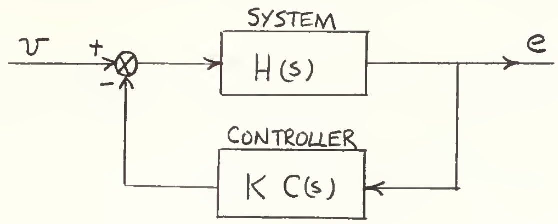


Fig. 4.1 System subject to disturbance

will cause the output or error to be zero (if the resulting system is stable) since

$$\frac{E(s)}{V(s)} = \frac{H(s)}{1 + K\,C(s)\,H(s)} \longrightarrow \frac{1}{K\,C\,(s)} \quad \text{as } K \rightarrow \infty$$

Unfortunately, in the ship motion problem infinite gain is impossible in the "controller", whose output is viewed as the force of the fins or rudder. This infinite gain would imply an infinite force output for a finite error, which is patently not the case. The real limitation for ship motion control is the peak force which can be exerted by the hydro-dynamic control surfaces.

From Newton[25], a theory exists which gives the optimum linear controller when its output is constrained to have a certain average power output. This appears superficially to be a valid approach, but actually it is forcing the physical problem to fit the theory, rather than vice-versa. It further limits the corrective force available instead of making intelligent use of the known limitation.

The designer of a ship pays a considerable penalty in space and weight for large fins and rudders and their associated high-power actuating devices. Because of this high price, he naturally desires to maximize the stabilization obtained from his installation, even at the expense of using quite sophisticated signal processing equipment, including perhaps many motion sensors and a digital computer. With this assumption, we must

discard our hopes for a completely linear theory since experience has shown[20] that non-linear control systems in general have much better performance in practice than linear systems, if the former are adequately designed.

If an error or the difference between the actual state of the ship and the desired equilibrium position exists, the control system will attempt to reduce this error by exerting corrective effort. It is intuitively appealing that this corrective action would be "better" at every instant if its magnitude were greater. That is, a maximum-effect, full-throw or "bang-bang" control system, which at every instant uses the maximum lift force possible at every hydrodynamic surface, will be postulated in the development to follow. A linear mode of operation in an arbitrarily small region of system error will be permissible, however. This full-throw type of control has been shown to be optimum in a class of problem which is concerned with the settling of a free system to equilibrium in minimum time.

4.3 A multi-dimensional approach for saturating controls

The design philosophy to be presented in this section was first given in Ref. 1, and is basically an extension of the so-called "Second Method of Lyapunov", a very general theory of dynamic behavior of linear and non-linear systems which is currently receiving considerable attention in the literature[26]. A summary of this approach is presented first, followed by a more complete development of the theory.

In general, we first attack this problem by specifying a quadratic criterion of instantaneous error for the ship, which could be, for example, the weighted sum of all the various squared displacements and velocities (ie: state variables) existing at a certain time. More formally we postulate an instantaneous measure of error with the quadratic form[18] $x^T Q x$, where x is the state vector of the ship from Section 2.2, and Q is a non-negative definite symmetric matrix.

In control, we are interested in the future as well as the present, and a measure of the future error of our instantaneous state is quite naturally given by the integral of the instantaneous error criterion over all future time. Since a random process is exciting the system, the expected value of future components of x is used in place of a determinate x. Also, future values of x are a function of the random process state variables, as well as those of the ship. By considering y to be the state vector of a linear system which includes both the ship and the random process generating model, the Second Method of Lyapunov is used to determine a symmetric matrix P, such that

$$y^T P y = \int_{\tau}^{\infty} dt \; x^T Q x$$

Thus, at every instant of time, a single number $y^T Py$ is a measure of all knowable future error, neglecting future values of control inputs. The various control signals are based on the requirement that the negative rate of change of this measure of future error be maximized continuously. The advantages of this approach are:

(1) The theory becomes more valid as the sea becomes rougher and the saturated outputs of the force transducers become relatively feeble with respect to the wave excitation.

(2) The design procedure is simple to execute and results in a completely linear system except for the saturation constraint.

(3) The resulting system is guaranteed to be stable.

(4) The multi-dimensional and inter-coupled aspects of the problem do not appreciably complicate the solution.

Now, to consider in detail the aspects of this procedure.

First, it is necessary to select the instantaneous criterion of error, $x^T Qx$. This is the area in which the skill and knowledge of the designer is applied with best effect. As in all optimization procedures, the critical and often least understood part of the design is deciding what is really wanted.

One rational choice is for Q to be a diagonal matrix with positive elements, λ_i. $x^T Q x$ would then be $\sum_i \lambda_i x_i^2$. The λ_i could be selected so that $x^T Q x$ is approximately proportional to the total energy of the ship. Since kinetic energy is represented by $\frac{1}{2}$ M V^2 and potential energy by $\frac{1}{2} K x^2$, with K an effective spring constant, the λ weighting terms could be selected proportional to the acceleration and displacement coefficients in the differential equations relating the velocity and displacement state variables, respectively, neglecting the cross-coupling.

Another potentially useful selection of the matrix Q would represent error by $\vec{X}_a \cdot \vec{X}_a + \mu^2 \vec{V}_a \cdot \vec{V}_a$, the squared disturbance displacement plus the weighted value of squared velocity in three dimensions, at a single point "a" on the ship. To obtain this measure easily, suppose that the vector $\vec{r}$ locates the point a from the center of gravity of the ship. If the vector $\vec{v}$ represents the instantaneous disturbance velocity of the center of gravity -- that is, the actual velocity less the mean forward velocity, the vector $\vec{\theta}$ represents the angular displacement (for example, the roll displacement ϕ would be a vector along x axis with a magnitude equal to ϕ in radians), the vector x is the disturbance displacement of the center of gravity, and the vector ω is the total angular velocity of the ship, then the motion of the point a is given by

$$\vec{x}_a = \vec{x} + \vec{\theta} \times \vec{r}$$

$$\vec{v}_a = \vec{v} + \vec{\omega} \times \vec{r}$$

for small angles of displacement.

Let the state of the ship be defined as in Section 2.2,

x_1 = Roll velocity, p
x_2 = Yaw velocity, r
x_3 = Sway velocity, v
x_4 = Heave velocity, w
x_5 = Pitch velocity, q
x_6 = Surge velocity, u
x_7 = Roll displacement, ϕ
x_8 = Yaw displacement, ψ
x_9 = Heave displacement, z
x_{10} = Pitch displacement, θ

Then, from evaluation of the vector equations,

$$
\begin{bmatrix} X_{ax} \\ X_{ay} \\ X_{az} \\ V_{ax} \\ V_{ay} \\ V_{az} \end{bmatrix}
=
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & -r_y & 0 & r_z \\
0 & 0 & 0 & 0 & 0 & 0 & -r_z & r_x & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & r_y & 0 & 1 & -r_x \\
0 & -\mu r_y & 0 & 0 & \mu r_z & \mu & 0 & 0 & 0 & 0 \\
-\mu r_z & \mu r_x & \mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\mu r_y & 0 & 0 & \mu & -\mu r_x & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} X_1 \\ X_2 \\ \cdot \\ \cdot \\ \cdot \\ X_{10} \end{bmatrix}
$$

where r_x, r_y, r_z locate the point of the ship to be stabilized from the ship center of gravity.

If

$$X_a] = \underline{R} \quad x]$$

Then,

$$\sum_i X_{a_i}^2 = x^T R^T R x = x^T Q x$$

and

$$Q = R^T R =$$

$$
\left[\begin{array}{c|c}
\mu^2 \begin{array}{cccccc}
r_y^2+r_z^2 & -r_x r_z & -r_z & r_y & -r_x r_y & 0 \\
-r_x r_z & r_x^2+r_y^2 & r_x & 0 & -r_z r_y & -r_y \\
-r_z & r_x & 1 & 0 & 0 & 0 \\
r_y & 0 & 0 & 1 & -r_x & 0 \\
-r_x r_y & -r_y r_z & 0 & -r_x & r_x^2+r_z^2 & r_z \\
0 & -r_y & 0 & 0 & r_z & 1
\end{array} & 0 \\
\hline
0 & \begin{array}{cccc}
r_y^2+r_z^2 & -r_x r_z & r_y & -r_x r_y \\
-r_x r_z & r_x^2+r_y^2 & 0 & -r_y r_z \\
r_y & 0 & 1 & -r_x \\
-r_x r_y & -r_y r_z & -r_x & r_x^2+r_z^2
\end{array}
\end{array}\right]
$$

Having determined, by any means, a measure of instantaneous error, $x^T Qx$, it is now necessary to evaluate the integral of this quantity over all future time. To do this, we use particular results from the Second Method of Lyapunov[26]. First of all, let us define the system under consideration. Fig. 4.2 illustrates the creation of the random forces and moments of the sea from a white noise driven model $\underline{G(s)}$. These forces excite the ship, $\underline{H^{-1}(s)}$, and a controller, sensing the motion, adjusts its

various control variables with full throw.

Section 2.2 showed how the matrix differential equation

$$\frac{d}{dt} x = A x + D_c F_c + D_W F_W$$

was formed for the ship alone. If the concept of system is enlarged to include $\underline{G(s)}$, which of course has its own state variables, then the differential equation of this new system can be written as

$$\frac{d}{dt} y = B y + D F \tag{4.1}$$

where y now contains, besides the ship state variables, the state variables of $\underline{G(s)}$. F is the excitation vector, having controlled force inputs and white noise as its components.

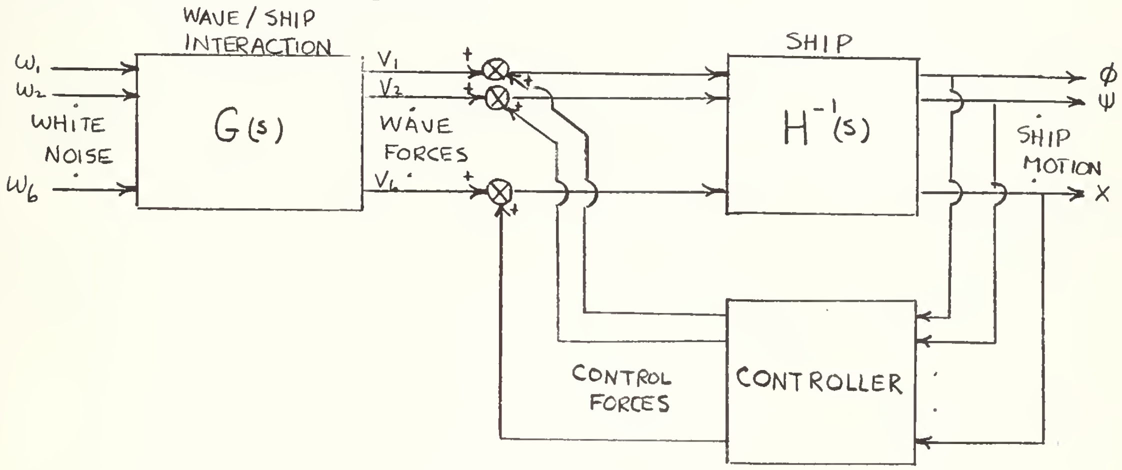


Fig. 4.2 The multi-dimensional control problem

If y has for its first ten variables the corresponding state variables of x, which is certainly allowable, the instantaneous error criterion $y^T Qy$ can be formed by using the Q matrix discussed earlier in the first ten rows and columns, and filling in the remainder of the matrix with zeros. This is equivalent to saying that we assign no instantaneous error to a state variable of the sea, since we cannot control it.

If the relationship

$$\int_t^{\infty} y^T Q y \, dt = y^T P y$$

is to be satisfied, then by differentiating the above equation we obtain

$$- y^T Q y = \frac{d}{dt}\left\{y^T P y\right\} = \left\{\frac{d}{dt} y^T\right\} P y + y^T P \left\{\frac{d}{dt} y\right\}$$

Now, in the computation of $\int_t^{\infty} y^T Q y \, dt$ we are concerned with the expected value of future y, disregarding future control action. Hence

$$\frac{d}{dt} y = B y$$

Substituting,

$$- y^T Q y = y^T B^T P y + y^T P B y = y^T (B^T P + P B) y$$

Therefore,

$$B^T P + P B = - Q$$

The elements of the symmetrical matrix P can be solved for numerically by the $\frac{n(n+1)}{2}$ independent equations in the above matrix equations, where n is the dimension of y.

The criterion to be used to fix the sign of the control variable is that the measure of future error, $y^T P y$, is to have the maximum possible rate of decrease at every instant. In other words,

$$\frac{d}{dt}\left\{y^T P y\right\} = \left\{\frac{d}{dt} y^T\right\} P y + y^T P \frac{dy}{dt} = 2\left\{\frac{dy}{dt}\right\}^T P y$$

is to have a maximum negative rate of change. But

$$\frac{dy}{dt} = B y + D F$$

Substituting,

$$\frac{d}{dt}\left\{y^T P y\right\} = 2 \left\{y^T B^T + F^T D^T\right\} P y$$

The effect of the control variables should be such that $F^T\left\{D^T P y\right\}$ is as negative as possible. This will obviously occur when F_i, the i^{th} control variable, is at its maximum, and has the opposite sign of the i^{th} component of $D^T P y$. That is,

$$F_i = - \left|F_i \max\right| \operatorname{sgn} \left\{D^T P y\right\}_i \tag{4.2}$$

where $\quad \operatorname{sgn} x = +1, \quad x > 0$

$\qquad\qquad\qquad\; = -1, \quad x < 0$

Eq. 4.2 is the desired control law.

The numerical coefficients of $-D^TP$ provides a weighting of the measured values of displacement and velocity in the various degrees of freedom of the ship, and also of the state variables of the random process generating model, $\underline{G(s)}$.

The measurement of these latter state variables poses an interesting problem. First of all, with the use of accelerometers, rate gyros, etc., it is feasible to measure the motion of the ship. Thus, the equations of motion of the ship, summarized by $\underline{H(s)}$, allow direct reproduction of the wave forces, F_w, by suitable weighting of the outputs of each of the motion sensors.

This six-dimensional random process will have been previously analyzed by statistical methods to obtain a matrix of power density spectra. Suppose that there is a small but finite noise in each of the signals representing the wave forces, which would certainly be the case in practice, and an approximation to the statistical nature of this noise is available (probably a white noise assumption) .

Even if there are steady state sinusoidal components of noise, which might occur with sustained vibration of the ship's structure, for example, this approach is still applicable and results in filters which tune out the offending sinusoidal components. By representing the input spectra of the wave forces as measured by $\underline{\Phi_{vv}(s)} = \underline{\Phi_{ss}(s)} + \underline{\Phi_{nn}(s)}$, where $\underline{\Phi_{ss}(s)}$ represents the signal or force spectra, and $\underline{\Phi_{nn}(s)}$ the unwanted noise spectra, then $\underline{\Phi_{vv}(s)}$ can be factored into $\underline{G(-s)}\ \underline{G^T(s)}$ by the methods of Ref. 1.

$\underline{G(s)}$ is separated, by partial fraction expansion, into $\underline{S(s)}$ and $\underline{N(s)}$ terms respectively, where for the assumption of uncorrelated white noise,

$\underline{N(s)}$ will be a numerical matrix. Then, the multi-dimensional feedback system shown in Figure 4.3 is constructed.

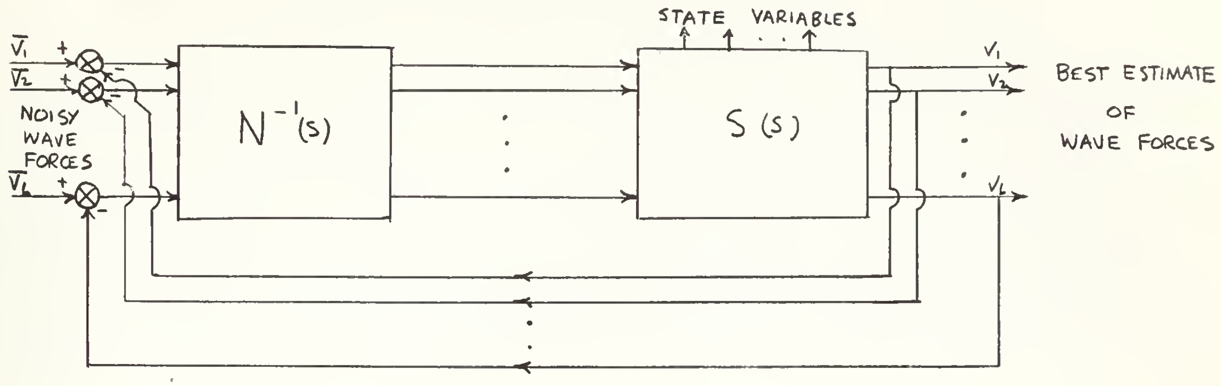


Fig. 4.3. Optimum multi-dimensional filter used to produce state variables.

This shows the configuration of an optimum multi-dimensional filter, as was discussed in Sections 2.3 and 2.4. In this application, the output or filtered wave force signals are not used, but the inner signals of $\underline{S(s)}$ are the desired state variables. Fig. 4.4 outlines the proposed scheme of control.

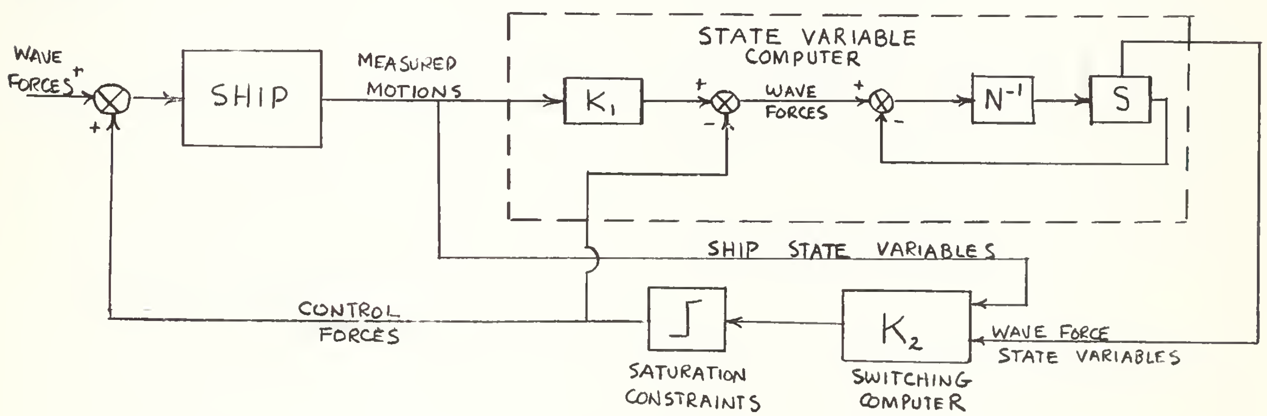


Fig. 4.4. A full-throw control system for multi-dimensional ship stabilization

Here, $\underline{K_1}$ weights the outputs of the motion sensors according to the equations of motion of the ship to calculate the external forces. $\underline{N}^{-1}$ and $\underline{S}$ act as an optimum filter to reproduce the state variables of the wave force random process. $\underline{K_2}$ weights the state variables according to Eq. 4.2,

with the matrix, $-D^T P$.

Broniwitz[30] has conducted an analog computer investigation of a particular scalar problem, stabilization of a second order system, using this theory. His results show a superiority of this approach as the system damping ratio becomes low and the ratio of maximum control force to r.m.s. disturbance force becomes small, compared with other "optimum" design procedures. As this domain of superiority is especially applicable to ship motion, considerable promise is offered for the multi-dimensional case.

Thus, to summarize, the multi-dimensional full-throw controller is made up of signals which it is practical to measure and performs essentially a linear computation, except for the binary nature of the output control commands. A fair amount of analog measurement and calculation is called for, but the basic problem itself is a very complex one, perhaps among the most difficult encountered in control systems engineering.

4.4 Some problems of practical multi-dimensional ship motion control

The design presented in the previous section is in a sense a centralized one -- it considers the ship as a whole and makes continuous decisions as to how each control variable should best be switched. In this section we will consider several problems which arise in the practical mechanization of this approach:

(1) Practical force transducers, such as hydraulically driven fins and rudders, require a finite time to reverse the direction of maximum force and cannot "switch" instantaneously.

(2) The actual hydrodynamic force exerted on a fin depends considerably on the relative direction of motion of the local water flow and not entirely on its mechanical displacement with respect to the ship.

(3) Practical motion sensors can be expected to deliver a signal

which is only an approximation to the true ship motion because of internal noise, cross-coupling effects, and local vibration levels.

In considering the first objection, we assume that the actuating equipment is sufficiently powered to allow full travel in a time equal to or less than the mean switching interval. If this assumption is not satisfied, then the system is rate limited rather than amplitude limited and the design premises are shaky indeed. In this case, certainly the capacity (ie: area) of the fins would not be efficiently used and either a reduction in their size or an increase in the power rating of the fin mover would appear to offer a more economically optimum design choice.

The output of the weighting operation $\left\{D^T P y\right\}_i$, it will be recalled from Section 4.3, is the multiplier of the i^{th} control force which determines the rate of change of the error criterion. When it has the value zero, obviously the force exerted at that instant contributes nothing to the reduction of this criterion and might as well be zero. That is, since a finite time is required to go from full up to full down, a near optimum choice of when to commence the downward travel would be so as to make the force pass through zero at the same time as $\left\{- D^T P y\right\}_i$ does. This would minimize the effect of the necessary small values of force required in a finite-time passage through zero by weighting them with small values of multiplier.

Thus, the signal for the i^{th} force variable to commence changing sign should be the prediction of sgn $\left\{- D^T P y\right\}_i$ at T_i seconds in the future, where $2\,T_i$ is the time required for full travel of the i^{th} fin. This will be given by sgn $\left\{- D^T P e^{B T_i} y\right\}_i$, where e^{BT_i} is the matrix exponential defined in Section 2.2, and B is given in Eq. 4.1. The switching computer K_2 shown in Fig. 4.4 would then have an i^{th} row of $- D^T P e^{BT_i}$.

The second problem, that of relative flow effects, is not so easily handled. The trouble is pictured in Fig. 4.5, which shows the lift breakdown of a typical hydrodynamic surface. Suppose that a feedback signal

is available of the actual lift on the fin, which is customary in contemporary fin installations. If the maximum force available and commanded is B, and

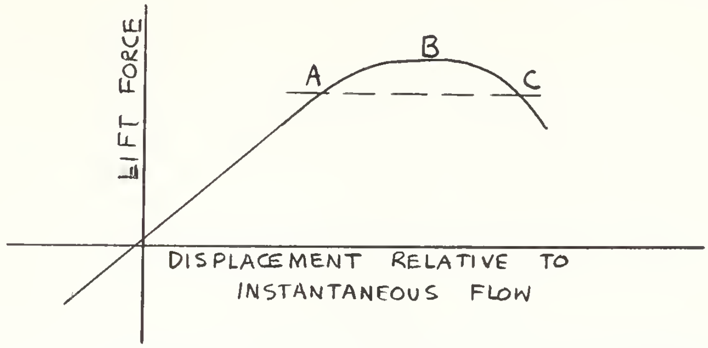


Fig. 4.5. Ambiguous force-displacement relation in fin breakdown

the actual force measured is given by the dashed line in Fig. 4.5, then it is not clear whether the relative displacement is at A or C, and the direction for corrective motion is unknown without (for example) additional pressure distribution measurements.

This problem is quite difficult, and typical of a general "peak-holding" class of control problem. While the mechanization of the solution to this interesting dilemma will not be elaborated upon further, one possibility would employ a small high-frequency sinusoidal "dither" on the fin -- or on a lift flap -- and the phase of the resulting sinusoidal measured force would distinguish between the case of point A and C. If the measured alternating force is in phase with the dither, then fin breakdown has not occurred, and, in fact, this sinusoidal force signal represents a valid error signal for a positioning control system for the fin since it goes to zero at the desired peak output.

The compensation of the noisy outputs of motion sensors is a standard problem of instrumentation engineering. If the noise can be statistically approximated by a power spectra, the theory in the previous section for optimum recovery of signal information is immediately applicable. Also, a convenient help in this respect would be the installation of sharply tuned filters at the instrument outputs which would reject sinusoidal components at dominant hull vibration frequencies excited by slamm-

ing, for example.

4.5 Adaptive measurement of wave force spectra

The key assumption in the previous theory is that the random process which represents the six-dimensional wave forces on the ship is stationary. This is, of course, not true, since ship course and speed changes vary the spectra as felt by the ship and since the properties of the sea change over a period of hours. However, the time variation of the parameters of the sea is very slow, so that the process can be termed a quasi-stationary random process and the previous theory is valid, providing that the current statistics of the sea are known.

There are three possible approaches to this problem:

(1) Since the statistics of the sea are not known, neglect them and their associated control theory, and carry out designs by more conventional (if less effective) means.

(2) Consider the worst case, statistically, which the ship could encounter (that is, excitation at the resonances of the ship) and design for this condition, accepting the degradation of performance at less severe sea states.

(3) Determine the dominant characteristics of the sea spectra, and attempt to measure them continuously and alter the control system to reflect these changes.

Some possibilities of the third approach are to be discussed in this section.

It is difficult to discuss intelligently the properties of the wave-force random process without experimental verification. However, there have been some statistical analyses of the sea which indicate considerable promise in adaptive measurement of wave forces.

Voznessensky and Firsoff[12] made an extensive series of measure-

ments of wave heights at sea using a wave pole, and performed auto-correlation analyses. They found that the correlation function was strikingly approximated in all cases by a damped cosine wave, $K_1 e^{-\alpha\tau} \cos\beta\tau$. From the discussions of Chapter II, it is clear that a generating model with the transfer function

$$\frac{K_2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

excited by white noise -- a standard second order system -- could be used to reproduce the process. The important result of these measurements was that the ratio of $\frac{\alpha}{\beta}$ remained essentially constant at .21, despite the wave-heights and directionality of the particular wave system measured. The constant ratio $\frac{1}{\sqrt{1 + \frac{\beta^2}{\alpha^2}}}$, is the non-dimensional damping ratio ζ in the standard form. This constancy appears to be a natural property of waves at sea, and tremendously simplifies the measurement of the spectra, as will be discussed.

The wave spectrum, if ζ is constant, is totally specified if the mean-square value of the signal and ω_n is known. The mean-square value of the signal is easily determined by a squaring and filtering operation of an electrical signal representing the process.

The ω_n value can be found through a process of zero-counting, or determining the average number of zero-crossings of the signal per unit time. From the work of Rice[27], Ehrenfeld, et al[28], and Steinberg[29] it can be shown that for a standard second-order system excited by white noise, the expected number of zero-crossings per second is $\frac{\omega_n}{\pi}$.

The possibilities of evaluating the dominant characteristics of a random process by squaring and averaging, and zero-counting, is a powerful tool for continuously evaluating the parameters of the sea. It is hypothesized that the dominant quantities of the multi-dimensional wave force spectra, since they arise from wave motion, can be measured by these

simple operations. As a simplified example, the effective frequency of excitation of the pitch-heave system, and that of the roll excitation, could be measured continuously by zero-counting the calculated forces as reproduced in Fig. 4.4.

The ability to make good statistical estimates of the parameters of the input random process without the need for a full-scale correlation or spectral analysis is a considerable boon to the control systems designer. It is felt that a valid approximation to the third alternative at the first of this section would be, on the basis of future experimental measurements of spectra at sea, to fix many of the parameters as sensibly constant and to measure some of the most important characteristics -- in particular, "frequencies" of excitation, ω_n -- and design a control system which would be nearly optimum in the sense of this chapter and which would be continuously "adapted" to changes in ship's course and speed and sea spectra.

4.6 Summary

The ship motion control problem is one of great complexity because of the inter-coupling of ship motion, the correlation between the various wave forces, and the saturation of control transducers. This chapter has presented one solution to this problem which has the virtues of near optimal performance, and the possibility of mechanization with a reasonable amount of computing equipment. Needless to say, only the germ of the idea appears here, and the real test of its validity must await accumulated measurements of ship force spectra and subsequent analog computer simulation of the control system.

CHAPTER V.

CONCLUSIONS AND RECOMMENDATIONS

5.1 Introduction

The penalty for making a frontal attack on such a complex problem as ship motion in a random seaway is necessarily a compromise between a desire to examine all facets in detail and a need to communicate the essential ideas clearly. This chapter will first attempt to outline the major viewpoints which have comprised the backbone of this thesis. Then, a specific program of proposed research to implement these ideas will be presented.

5.2 Major ideas

The basic idea which threads through all past chapters is that the random wave forces and moments on a ship at sea are the dominant quantities of interest in any study of the complicated behavior of multi-dimensional ship motion. With the development of suitable theory in Ref. 1 to analyze the statistical interrelations among these forces and moments, all useful information regarding (1) average motion levels , (2) short term motion prediction, and (3) optimum control decisions can be found. In short, a path is now opened which will explain all that can be explained about ship motion within the limits of current mathematical knowledge and, what is perhaps more significant, the needed measurements can be easily made with standard instrumentation and data reduction methods.

Chapter II has briefly shown that the ship can be described in the two basic models for linear systems -- the matrix transfer function in Laplace transform notation and the matrix differential equation, the latter using the significant concept of "state". The idea of a white noise driven model has been introduced, and the statistical measures of power density

spectra have been interpreted as means for determining this linear model.

The practical problem of approximating experimental spectra with rational Laplace transforms has been examined in Chapter III. Besides presenting some graphical aids in this task, it has been demonstrated that methods now in use for statistical data reduction using a digital computer introduce unnecessary error in the final spectra.

Also, it was shown that a considerable degree of statistical correlation exists between the roll and pitch motion of a ship at sea, the major portion of which is hypothesized to arise from the correlation between the respective wave excitation forces.

Chapter IV has developed in some detail a proposed scheme of simultaneous control of many force transducers to stabilize a ship at sea. With the virtues of near-optimality, full-throw control, case of instrumentation, stability, and consideration of all interactions in the ship motion and the input forces, this scheme offers considerable promise of extracting near the maximum in stabilization from fins and rudders.

Preliminary analog computer investigation reported elsewhere[30] gives substantial support to this design concept for ship control. The peculiar problems of relative water velocity effects, hydraulic rate saturation, and adaptive spectral measurement have been discussed.

5.3 A proposed research program

The following is a proposed program of needed and unified basic research in the areas of ship behavior at sea and ship motion control.

A. Ship parameter measurement

Determine, experimentally and analytically, the significant coefficients of the linear differential equations (as a function of forward velocity) which govern the total motion of the particular ship type under

investigation. Use free or constrained models with a suggested greater use of initial condition response. Also, statistical analysis of measured ship motion at sea will yield useful verification of the small-scale results.

B. Wave force measurement

On a ship at sea, or on a free model in a random wave generating facility, make sufficient weighted motion measurements to infer -- through the assumed linear differential equations of motion -- the wave force or moment excitation in each of the six degrees of freedom. An alternate experiment could make use of a completely rigid model in a random wave tank, with the desired forces measured directly.

C. Immediate wave force analysis

From the six-fold record of wave forces, conduct a complete cross-spectral analysis, either by digital computation of correlation functions and subsequent transformation, or by direct analog measurement. Approximate these spectra by rational transforms. Factor, preferably with a digital computer, the matrix of power density spectra with the method introduced in Ref. 1. The result, a 6x6 matrix of transfer functions, completely characterizes the random nature of the sea and can be immediately used in analog computer studies.

D. Extended wave force analysis

Collect a number of matrix descriptions of the sea. Attempt to determine relatively invariant parameters, as well as the most significant varying ones. Define a typical or design sea, possibly on a worst-case basis for the ship under consideration.

E. Relation to wave-height directional spectra

Conduct theoretical and experimental analyses of the relation

between the observed statistical properties of wave forces on a ship or model and the directional spectra which characterizes the sea.

F. Multi-dimensional control

Using the matrix model developed under steps C and D above, conduct analog computer investigations with the following objectives:

(1) To evaluate the control theory outlined in this thesis, as well as to stimulate other approaches to the multi-dimensional stabilization problem.

(2) To determine the importance of using knowledge of the input random process as an aid to control, as well as the penalties associated with imperfect or no knowledge of it.

(3) To determine the optimum balance between large fast power actuators of rudders and fins and large surface areas of these appendages, and also the reduction in stabilization because of rate saturation and lift breakdown.

G. Ship motion analysis

From accumulated knowledge of wave-force spectra, determine the significant parameters of the ship which affect seaworthiness and emphasize undesirable motion.

H. Adaptive measurement of seas

Conduct analytical and experimental investigations into adaptive measurements or "tracking" of significant parameters of random processes, with the goal of providing a shipboard control system with a self-adjusting capability as sea conditions change.

5.4 Conclusions

The above program, which unifies the central themes of this thesis,

is believed to offer a promising and practical roadmap into the vital areas of ship motion and control. Many problems and an unbelievable amount of data remain to be analyzed and digested, but no valid approach to such a complex problem as this can avoid such a large-scope investigation.

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